CATFISH: LYING IN MATCHING MARKETS WITH CHEAP TALK

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Abstract. We study a search model of online dating with nontransferable utility where agents are vertically differentiated, self-report quality, and must go on costly dates to verify a match's quality. We show that these per-date costs induce some agents to over-report their type, consistent with the stylized facts of online dating platforms where users frequently over-report characteristics like height and income, a phenomenon known as catfishing. This make agents less picky by preventing high types from rejecting some low types, and since externalities in matching markets without transfers can make agents inefficiently picky, these costs can improve total market surplus. A monopolist platform owner may also have an incentive to increase per-date costs in order to increase profits. Thus, inducing lying amongst users can actually be optimal for a platform.

Keywords: Costly Verification, Non-transferable Utility, Cheap Talk, Search and Matching, Assortative Matching, Online Dating

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1. Introduction

This paper studies matching markets with "catfish", a neologism for someone who attempts to attract matches on an online dating platform by lying about themselves. Online dating has become a common component of dating and partnership formation, and is a fast growing market, taking in \$1.08 billion in revenue for dating sites and \$572 million for dating apps in 2014.[2] Lying is an important factor in search on these platforms; the distributions of reported types for traits like income and male height tend to be shifted right on online dating platforms relative to the broader population, suggesting misreporting,[3] and one industry study found that 20% of women and 33% of men admit to lying on their online dating profiles.[4] This report also offers advice on how to account for such misreporting, advising women to "assume the men you meet might not be quite as tall, as successful or as connected as they say they are, and then decide whether you'd still consider dating them regardless", suggesting that misreporting is an important factor in agent search strategies. We study a stylized model motivated by this issue, with a two sided (men and women) platform allowing agents to search for matches on the other side[5] where agents can misreport their type.

Specifically, we model a one-to-one nontransferable utility (NTU) matching market with random search[6] and time discounting where agents pursue long term (permanent) matches. Agents are vertically differentiated,[7] are distributed over a continuum of types, and self-report type. When agents meet, they see one another's reports and choose whether to enter a type verification phase (date) or to part and return to search. If they go on a date, they see each other's true type. They then decide whether or not to match permanently.

Going on a date is costly, allowing some agents to profitably lie. Before a date, agents weigh the expected payoff from continued searching against the benefit of matching today less the cost of the date, accepting a match if the latter is higher. However, after the date

[2]http://www.wsj.com/articles/the-dating-business-love-on-the-rocks-1433980637
[3]http://blog.okcupid.com/index.php/the-biggest-lies-in-online-dating/
[4]http://www.ayi.com/dating-blog/ayi-top-online-dating-profile-lies/
[5]This analysis can easily be extended to a same-sex dating market.
[6]Over time, agents receive random draws from the set of agents on the other side of the market.
[7]Each type is characterized by a level of quality, and all agents prefer higher quality matches to lower quality matches.

this per-date cost is sunk and drops from the agent's decision, making them less selective. That is, once you've already made the effort to meet and learn about someone you should be less picky, since if you start your search over again you'll have to make a costly investment in learning about the next person. Thus, some agents will be rejected for a date if they truthfully report their type, but, if they get a date by pooling with more attractive peers, they will be accepted after the date when standards are lower.

We focus on an equilibrium with a minimal amount of pooling, which we term the "Limited Pooling" equilibrium. In this equilibrium, agents self partition into "classes", where each agent only matches within their class. Classes will typically be larger–and thus agents less choosy–when per-date costs are higher, since these costs allow lower quality agents to pool with and match to higher types who would like to reject them ex-ante.

We study how a strategic platform charging a single fixed fee[8] can utilize per-date costs to improve firm profits or maximize surplus as a social planner. We first consider the case where per-date costs are prices (for example, a price to communicate with an individual you're interested in). Externalities endemic to NTU search markets make agents too picky. In particular, agents don't care about their match's payoff, or the payoff of other agents in the market, only their own. Without transfers, other agents have no way of making them internalize these costs. Thus agents will chase after high quality matches, ignoring the fact that if they get a high quality match, someone else must get a low quality match, so their gain is another's loss. Thus the social planner will want agents to be less picky, since choosy behavior results in surplus loss due to time discounting and benefits the social planner much less than it does the individual. Thus, per-date costs can counter this excessive pickiness, and we find that a social planner will utilize positive per-date prices to make agents less picky and increase total surplus.

[8]We discuss a schedule of fixed fees in Appendix 6.1.1.

A profit seeking monopolist[9] platform will also charge positive per-date prices.[10] In this environment, the platform fully extracts surplus from the lowest quality agent who joins, while leaving rents to all higher class agents due to the lack of price discrimination. By forcing higher types to match to lower types they'd like to reject, per-date costs can be interpreted as inducing a transfer from high types to low types, which allows the platform to extract more surplus from agents by making the indifferent agent better off (and willing to pay more) at a cost to the rents of high type agents, which the platform does not value.

We also consider the case where per-date costs are frictions, and a platform has access to technology that can lower per-date frictions at a cost to the platform. Some online dating platforms in fact offer technologies such as video-chat and ID verification to lower informational frictions. Even in this case, we find that a monopolist platform often prefers high per-date costs, despite their great cost to agents. Per-date costs can again be interpreted as inducing a transfer from high types to low types, increasing the fixed fee the platform can charge. With appropriately chosen per-date costs, this benefit offsets the direct surplus losses due to per-date frictions. Thus, in this environment platforms often do not have an incentive to induce truth-telling or informative reporting, and in fact may benefit from inducing lying if lowering frictions is even slightly costly. A social planner will generally prefer to lower per-date frictions to zero if the cost to the platform of this technology is sufficiently low, but there are special cases where even a social planner will prefer higher per-date frictions.

In the broader set of equilibria, the ability to freely report type allows for many forms of pooling, including highly non-monotonic reporting where, for example, agents in a high type class and a low type class make the same report while agents in an intermediate quality class make reports that uniquely identify their class. However, we find that, regardless of reporting strategies, equilibria are characterized by a class partition where, after going on

[9]A platform with significant market power may be a reasonable approximation of the online dating market as the market is fairly concentrated and the fact and much of the market outside of the two largest firms, IAC and eHarmony, is highly differentiated niche platforms like JDate and ChristianMingle.

[10]Explicit per-date prices are currently uncommon in dating platforms, though some platforms, like Ashley Madison and It's Just Lunch, have utilized them. More common are contracts that limit the number of contacts one can make in a given period of time and charge a premium for unlimited contacts, which has qualitatively similar effects but is less tractable to model.

a date, agents will only match to draws whose true type is within their class. Hence, this paper contributes to the literature by extending the coarse, class based form of positively assortative matching (PAM)[11] found previously in NTU search models like Macnamara and Collins (1990) and Burdett and Coles (1997) to an environment with cheap talk and costly type verification.

It also contributes to the literature on two-sided platforms–specifically, the literature on strategic matching platforms. In particular, we show how informational frictions can be profitably used by a strategic platform to counteract externalities in matching markets. A recent survey of the search and matching literature by Chade, Eeckhout, and Smith (2015) identified the nature and implications of externalities in matching markets as one of the major open questions in the field, so this analysis addresses an important hole in the literature.

We'll now describe some additional salient features of the online dating market. It features significant concentration, with eHarmony taking in \$310 million in revenue in 2014 and platforms owned by the IAC (Match, Tinder, OkCupid, etc.) taking in \$601 million, almost all of which was from dating websites, where total dating site revenue was \$1.08 billion. [12]. Membership is slightly less concentrated, with IAC platforms serving 21% of users in 2014 and eHarmony serving 13% [13]. The remainder of the market is composed of small platforms, many of which are niche dating sites, and recent dating app entries. Platforms typically either offer a single service at a positive price, or a free, ad-supported service and a premium paid service with additional amenities, such as unlimited messaging, better search options, and video chat. Many platforms engage in second degree price discrimination, offering significant discounts for longer contracts. Typical contract lengths are 1, 3, 6, or 12 months. Few major platforms engage in overt demographic based price discrimination, although Tinder prices based on age.

While this paper focuses on an application to online dating markets, matching markets with search and costly type verification–and thus an incentive for lower quality agents to

[11]Higher types match with higher types, lower types match with lower types.

[12]http://www.washingtonpost.com/news/business/wp/2015/04/06/online-datings-age-wars-inside-tinder-and-eharmonys-fight-for-our-love-lives/

[13]http://www.wsj.com/articles/the-dating-business-love-on-the-rocks-1433980637

pool with higher quality agents–appear in a variety of contexts, notably in job search. While the NTU assumption is less palatable for job search, as wages are often bargained over and can be set flexibly by the employer, there are also often limits to the degree of transferability. Among other things, firms may have their wage setting abilities constrained by regulations like minimum wages, and progressive taxation makes larger transfers more costly. Hence, the efficiency and profitability of positive per-date frictions we find assuming NTU may extend–perhaps with some attenuation–to partially transferable utility (PTU) environments that may more credibly model applications like job search. Additionally, in some job search applications like the market for medical residents, wage offers are extremely compressed due to the market structure, making NTU a reasonable assumption. While the National Resident Matching Program ends with a one-shot assignment game based on rankings reported by each side, hospitals and students meet in time consuming and costly interviews before reporting their preferences, suggesting a search model like the present may capture some stylized characteristics of such markets.

The remainder of this paper is organized as follows: Section 2 discusses the related literature and this paper's place within it. Section 3 lays out the basic theoretical framework this paper uses, characterizes the set of equilibria in this environment, and provides equilibrium selection arguments. Section 4 incorporates a strategic platform that can change the magnitude of the per-date costs. Section 5 concludes. Section 6 is the appendix, which includes proofs for many of the propositions in the paper, as well as analysis of the model with alternative assumptions, such as different matching technologies.

2. Related Literature

This paper follows a rich literature on search and matching. In particular, it fits within the literature on search and matching with NTU. McNamara and Collins (1990) first studied the NTU search environment with a continuum of types and found the distinctive partition or class equilibrium common in this literature, where agents in each class only match within their class. Burdett and Coles (1997) extended this analysis to a steady state environment with exogenous inflows of agents and endogenous outflows. The present paper is closely related to this strand of the literature. In particular, the constant returns to matching model can be interpreted as Burdett and Coles with an additional reporting stage. Eeckhout (1999) extends this result to multiplicatively separable preferences, and Chade (2001) extends it to fixed search costs. Smith (2006) looks at even more general preferences, situating the partition result in a larger class of equilibria where partitioning does not necessarily hold. There is a parallel literature for the transferable utility (TU) assumption, with Shimer and Smith (2000) studying equilibria in the analogous TU environment. Generally, TU makes characterizing equilibria, payoffs and agent behavior more difficult.

There is also a small literature on strategic matching platforms with search. Bloch and Ryder (2000) study a monopolist platform environment that can offer frictionless NTU matches for a fixed fee or a fixed proportion of match surplus, with an outside option of NTU search. Damiano and Li (2007) study vertically differentiated agents and a monopolist that creates a continuum of platforms and sets prices to induce agents to join their assigned platform and match with identical agents. Given the simplicity of observed contracts in this market and the potential frictional costs of partitioning agents into many small platforms, our paper instead considers what a less ambitious platform can do in an environment where draw rates proportional to the mass of agents on a platform make infinite partition of the space into measure zero platforms inefficient. Damiano and Li (2008) study competition between matching markets.

This paper also relates to the literature on cheap talk and information transmission. The cheap talk literature was pioneered in Crawford and Sobel (1982). Applications of cheap talk, signaling, and information transmission to matching markets include Hoppe et al. (2009) and Hopkins (2012), who study matching with signaling. Bilancini and Boncinelli (2013) address a similar NTU environment but assume only one side has unobservable types and consider a binary choice between type certification and full information matching and hiding one's type and matching randomly. The present paper differs from these works by focusing on a cheap talk environment. Ko and Konishi (2010) study a profit-maximizing platform matching firms and workers in a many-to-one environment, where firms and workers report match-specific wage offers and desired wages, respectively. They find that manipulating reporting by curtailing the message space can improve profits. A current working paper, Hagenbach et al. (2015), studies an environment very similar to ours, with an initial reporting stage and a costly type verification stage before permanent matching in a search environment with vertical differentiation and NTU. However, they consider a two point type distribution, while our analysis focuses on the class structure that only appears non-degenerately in a model with a continuum of types. We also study strategic platforms, while they focus on a nonstrategic platform.

This paper also relates to the literature on two-sided platforms, pioneered in Rochet and Tirole (2003, 2006) and Armstrong (2006). More recent work includes Weyl (2010), Bedre-Defolie and Calvano (2013), and Lee (2013). In contrast to the majority of this literature, which takes advantage of simple, exogenous specifications for network externalities, this paper explicitly models the special case of network effects induced by a search model of matching.

3. Model

3.1. **Preliminaries.** We model a heterosexual market on an online dating platform and denote the two sides men (m) and women (w). Agents are characterized by a single vertical characteristic representing quality or attractiveness, where every agent strictly prefers higher quality matches. Quality for side j is distributed over $[\underline{q}_j, \overline{q}_j]$, $\underline{q}_j > 0$.[14] When agents join the platform, the platform solicits a report on their true type $\hat{q} \in [\underline{q}_j, \overline{q}_j]$, representing their online dating profile. On the platform, agents engage in bilateral search for partners in continuous time, with a discount rate of $r \in (0, \infty)$. Agents receive random draws from from the endogenous distribution of agents on the other side $G_j(q, t)$ according to a Poisson process with arrival rate α, where G_j is continuous[15]

[14] If $\underline{q} = 0$ there can be infinitely many classes, which poses difficulties for certain aspects of the analysis.

[15] We'll prove this later.

When they meet, each agent observes the other's report, and they make an ex-ante decision whether or not to propose a date. If both propose a date, they pay a per-date cost,[16] learn each other's true type, and make an ex-post decision whether to propose a match. If both propose a match they marry forever. If either rejects in the first stage they part costlessly, while if either rejects after the date they part having paid the per date cost. Inflow into the platform is exogenous and time invariant, with the cumulative distribution given by $F_j(q)$, where F_j is twice differentiable and has full support on $[\underline{q}_j, \overline{q}_j]$. The corresponding density is given by $f_j(q)$. The total mass of inflow is equal for both sides and normalized to 1. Outflow is determined endogenously by the rate of acceptances and the mass on the platform. The mass of agents on the platform is given by N.

Total match surplus is given by a function $u(q_m, q_w) \equiv \psi_m(q_m)\phi_m(q_w) + \psi_w(q_w)\phi_w(q_m)$, where each agent's payoff is multiplicatively separable into an own type component ψ and a match's type component ϕ. Both are weakly increasing twice differentiable positive functions, with ϕ is strictly increasing. We assume non-transferable utility (NTU), where $u_w(q_m, q_w) \equiv \psi_w(q_w)\phi_w(q_m)$ and $u_m(q_m, q_w) \equiv \psi_m(q_m)\phi_m(q_w)$. This means that agents cannot bargain over the apportionment of surplus, perhaps due to social norms, which may be plausible in some matching markets such as dating markets. NTU, along with the multiplicative separability of each agent's own type and match type in their payoff, ensures the very simple and tractable class structure common to this literature. Per-date cost for an agent of type q_j on side j is $\psi_j(q_j)c$. This is a strong assumption, but it is also necessary to preserve the class structure of the equilibrium.[17] When $\psi = 1$, as in the commonly assumed case where utility is simply match's type, per-date costs can be thought of as either a price imposed by the platform or the opportunity or effort cost of going on a date. When $\psi \neq 1$, per date costs should be thought of as opportunity costs that are increasing in type. [18] Unless otherwise

[16]We assume agents cannot match sight-unseen. This seems consistent with most marriage/partnership formation, where some amount of quality verification precedes commitment. Even arranged marriages typically involve reconnaissance by relatives.

[17]We'll relax this assumption in numerical simulations in Appendix 6.1.3.

[18]Note that, if $\phi_w = \phi_m$, we can define a new trait $x' = \phi(x)$ and find a distribution H such that $F(\phi^{-1}(x')) = H(x')$. Thus, when we assume symmetric payoffs it will suffice to consider $\phi(x) = x$. However, when we make distributional assumptions we must note that they will change when types are mapped back into the original distribution.

noted, agents have an outside option of zero, such that every possible match is preferable to remaining unmatched.

We'll focus on stationary equilibria where

- **Assumption 1 (STN)** : Each agent believes $G_j(q,t) = G_j(q)$.

Further assume stationary agent strategies, where a strategy is an agent's ex-ante date decision for each reported type and ex-post match decision for each true type. Let $\mu_i(\hat{q})$ be agent i's belief about the distribution of q given a report $\hat{q}$. Following Burdett and Coles (1997) and extending the definition to a game of incomplete information, we utilize the following definition:

Definition 1. Given (G_m, G_w), a Bayesian perfect partial equilibrium (BPPE) is a strategy profile and beliefs μ where STN is satisfied, agents maximize utility subject to their belief about other agents' types and actions and follow sequential rationality, and beliefs are consistent with Bayes rule wherever possible.[19]

This definition identifies a set of candidates for a steady state equilibrium, which we will later winnow down by requiring that inflows equal outflows.

We can now establish several useful properties of the agents' strategies and payoffs. First, we'll establish that agents follow cutoff strategies:

Lemma 1. *In a BPPE each agent x accepts all draws above some cutoff $\underline{q}_x$ ex-post and rejects all draws below. Each agent x accepts all draws with expected discounted match quality above a cutoff ex-ante and rejects all draws below. Each agent accepts a strictly positive measure of ex-post matches in equilibrium.*

Proof. After true type is revealed, an agent chooses between a continuation value independent of current draw and the value of the draw. Accepting is costless, so they must accept if the utility of the draw exceeds the continuation value and reject otherwise. Before true type is

[19]Also assume agents reject ex-post matches when indifferent and accept ex-post matches they strictly prefer to continued search when they believe the probability of beign accepted is zero.

revealed, agents choose between a continuation value independent of the current report and the expected discounted match quality associated with that report.

Accepting a zero measure of dates or matches yields a continuation value of zero. Accepting any agent yields a strictly positive payoff ex-post, thus agents must have a cutoff below the maximal type on the other side. □

Let $U_w(q|\hat{q})$ denote woman $q's$ expected discounted lifetime utility when reporting $\hat{q}$. $q_{wE}(q|\hat{q}) \equiv U_w(q|\hat{q})/\psi_w(q)$ is then the expected discounted match quality. Given symmetric definitions for men, we can easily show that higher type agents get matches of weakly better discounted quality (and thus higher utility) and that agents with higher quality matches must have weakly higher types:

Lemma 2. *If an agent is of type $x > x'$, $q_{jE}(x|\hat{q}_x) \geq q_{jE}(x'|\hat{q}_{x'})$. If $q_{jE}(x|\hat{q}_x) > q_{jE}(x'|\hat{q}_{x'})$, $x > x'$.*

Proof. Given Lemma 1, any agent that will accept x' ex-post must accept x and x can always mimic the x' strategy. Thus an x agent can always obtain at least as high expected match quality as an x' agent. □

We will make one of two assumptions about the rate of draws agents face. Specifically, we'll assume they face *linear returns to matching* (LRM) in the main body of the paper, and consider the case of *constant returns to matching* (CRM)

[20] in Appendix **??**.

- **Assumption 2A (LRM)** : Agents receive a rate of draws α proportional to the mass of agents on the platform, normalized to N.
- **Assumption 2B (CRM)** : Agents receive a constant rate of draws α, normalized to 1.

[20]Note that LRM is sometimes referred to as a quadratic search technology, owing to the quadratic nature of the total number of draws in the market as a function of the number of agents, and CRM is sometimes referred to as a linear search technology based on the linear rate of total draws in the market as a function of the total number of agents.

Linear returns to matching means that the frequency of draws is proportional to the mass of agents on the platform and that thick markets make search faster. Most past work on search with NTU has focused on the constant returns to matching environment, and this may be more appropriate for traditional forms of partner search where finding potential matches is time consuming and these frictions put an upper bound on the number of draws an agent can consider, regardless of the size of the market. However, on an online dating platform, we'll argue that linear returns may be more realistic. With easy search, filtering, and detailed information available with a single click, it's plausible that more agents on the platform means more draws, since one may quickly exhaust a small list of potential matches by paring it down to a handful of likely matches.

In fact, the linear returns environment is significantly more tractable than the constant returns one: since there are no per-draw costs in this model, having to eliminate more agents outside of your acceptance region imposes no cost, and thus changes in the mass of agents outside your class has no effect on your optimization problem. This can be motivated by the nearly costless filtering out of undesired matches that may be achieved on a search platform. With CRM, by contrast, more agents in other classes means it will take longer to get a draw from your class, making behavior in each class dependent on behavior in every other class.

Define $\lambda \equiv Pr[match|q,\hat{q}]$ and $\gamma \equiv Pr[date|q,\hat{q}]$. Given that the continuation value of a woman of type q reporting $\hat{q}$ (and the case for men is symmetric) is their lifetime expected utility, $U_w(q|\hat{q})$, and is also equal to the ex-post cutoff draw $q_{wl}(q,\hat{q})$, the dynamic program for this environment gives us the following optimization condition ex-post for a small time period dt:

$$U_w(q|\hat{q}) = \frac{U_w(q|\hat{q})(1-\lambda\alpha dt) + \lambda\alpha dt E[\phi_w(q')\psi_w(q)|q,\hat{q},match] - \gamma\alpha dt\psi_w(q)c}{(1+rdt)} \tag{3.1}$$

Where c does not appear on the left-hand side (LHS) because the per-date cost has already been paid when the ex-post match decision is being made. Taking the limit as $dt \to 0$, we have

$$U_w(q|\hat{q}) = \frac{(\alpha\lambda E[\phi_w(q')|q,\hat{q},match] - \alpha\gamma c)\psi_w(q)}{\lambda\alpha + r} \tag{3.2}$$

and applying the equality between continuation and cutoff acceptance utility,

$$\phi_w(q_{wl}(q,\hat{q}))\psi_w(q) = \frac{(\alpha\lambda E[\phi_w(q')|q,\hat{q},match] - \alpha\gamma c)\psi_w(q)}{\lambda\alpha + r} \tag{3.3}$$

Notice that this specification for the cutoff means that the way one's own type enters the utility function doesn't affect agent behavior, and thus only matters when one considers welfare or adds prices to the model.

Because the continuation value is the same for both the ex-post and ex-ante decisions, but ex-ante the per-date cost is not sunk, the ex-ante cutoff is simply $\phi_w^{-1}(\phi_w(q_{wl}(q,\hat{q})) + c)$ if type is certain or expected discounted match quality equal to the same.

Lemma 3. *Ex-post cutoffs and optimal strategies are independent of ψ_j.*

Proof. Direct inspection of 3.3. □

3.2. **The Set of Equilibria.**

3.2.1. *Ex-Post matching structure.* We can now analyze ex-post matching behavior. While the structure of reporting can be quite complex, ex-post matching behavior is simple and highly consistent with previous research on NTU search models with observable types. In particular, Proposition 1 shows that agents will partition themselves into classes in equilibrium. First, we'll formally define the terminology:

Definition 2. We'll call an interval of types a *class* if every agent with a type in that class accepts any type from that interval ex-post and only forms ex-post matches with types within the class. Define the lower bound of a class n on side j $q_j(n)$.

Note that this is a condition on the second stage where type has been revealed. Agents in a given class may reject agents within their class ex-ante, and accept dates from agents outside their class depending on the reporting structure.

Proposition 1. *The distribution of agents on each side is partitioned by intervals (or classes) of agents where, for each class n, men(women) in class n will accept any woman(man) in their corresponding class n ex-post, will reject any woman(man) below class n, and will be rejected by any woman(man) above class n.*

Proof. Consider a $\overline{q}_m$ man. $\overline{q}_m$ men are accepted by every woman ex-post and must accept women above a cutoff strictly below $\overline{q}_w$ by Lemma 1. Thus, there is a nontrivial interval over which every $\overline{q}_m$ man accepts matches ex-post. Denote the lower bound $q_w(1)$ for the highest type man's cutoff type (in the distribution of women) and $q_m(1)$ for women. Then by Lemmas 1 and 2, every agent must accept this interval ex-post as they have lower types. Since women will accept men above $q_m(1)$ ex-post, a man above that type can mimic any man's strategy. Thus, every man above $q_m(1)$ will get the same payoff in expectation and thus the same cutoff $q_w(1)$. We'll call the interval of women $(q_w(1), \overline{q}_w]$ class 1 of side w and denote the nth class class n. A symmetric analysis yields $(q_m(1), \overline{q}_m]$, class 1 of side m. We can proceed inductively from here. Given that every woman above $q_w(n)$ rejects any man at or below $q_m(n)$, $q_m(n)$ type agents face a problem analogous to $\overline{q}_m$ men, and accept every woman in an interval whose lower limit is defined as $q_w(n+1)$. Similarly, every lower type man accepts all women in $(q_w(n+1), q_w(n)]$. Thus every woman in class $n+1$ is accepted by the same set of men and must have the same payoff in expectation and thus the same cutoff $q_m(n+1)$. A symmetric analysis shows that every man in $(q_m(n+1), q_m(n)]$ only accepts women above $q_w(n+1)$.

□

Figure 3.1 shows the class structure of an equilibrium. Agents in each class only match to agents in the same class on the opposite side. An agent in class 3, for example will accept anyone in a classes 1, 2, or 3 ex-post, and will be rejected ex-post by every agent in classes 1

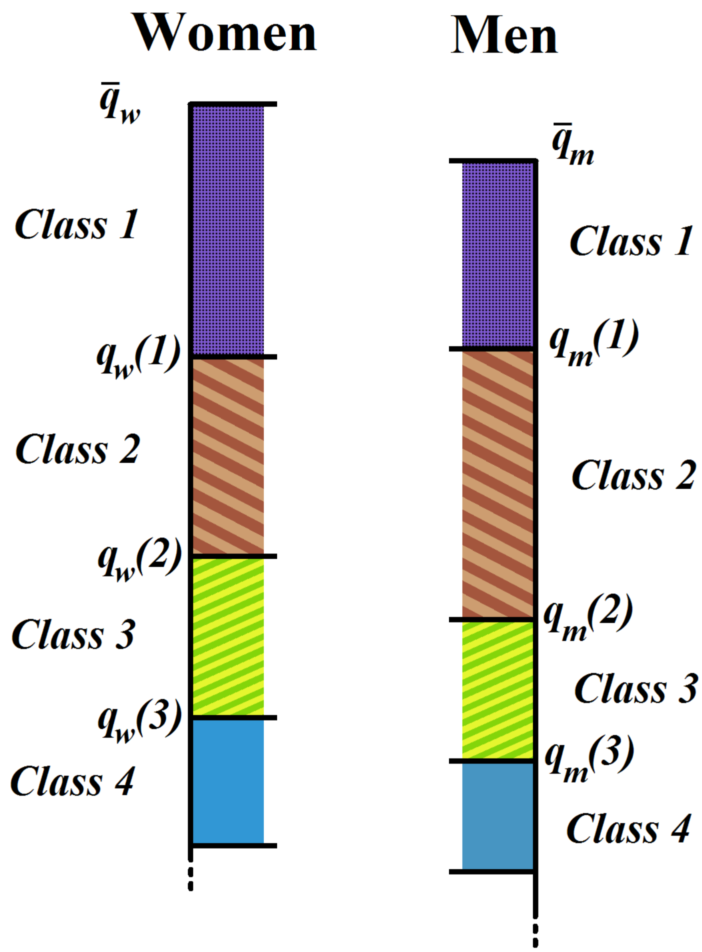

FIGURE 3.1. The class structure of an equilibrium.

and 2. Note that the length of classes can vary based on density and the class cutoffs need not be symmetric if the distributions are not. This class structure ensures that agents in each class must have a higher payoff than agents receive in any lower class.

Lemma 4. *Suppose type q is in class m and type q' is in class n. If $m>n$, $q_{jE}(q) > q_{jE}(q')$.*

Proof. Suppose not. Then the q and q' agents receive the same payoff and thus must have the same ex-post cutoff. But then they are in the same class. Contradiction. □

3.2.2. *Reporting Structure.* We can now address the reporting stage of the game. While the ex-post class structure was simple, there are a wide range of possible reporting structures,

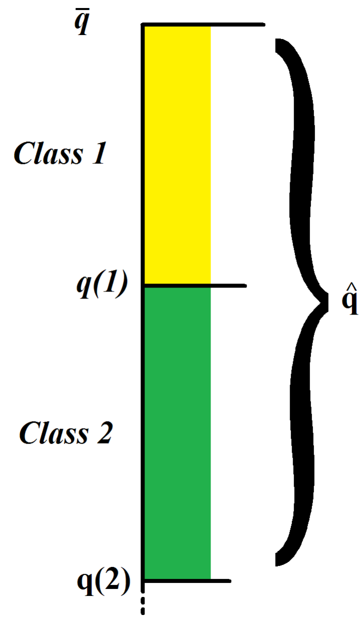

FIGURE 3.2. This diagram shows inter-class pooling, as in Proposition 2.

including pooling over multiple classes. Because of the bilateral nature of the signaling, it's important to note the possibility of asymmetric reporting where, for example, every woman pools and every man reports their class. The redundancy of the two sided reporting and acceptance decisions means that, so long as one side reports their class and accepts everyone ex-ante, the other side can simply accept only the the class they will match to ex-post.

Definition 3. On a given side, we'll call an interval a *contiguous pool* if there is a report $\hat{q}$ such that, for every type in the interval, some agent of that type reports $\hat{q}$ or another report that is accepted by the same set of agents ex-ante.

Pools, then, are defined based on reporting and the ex-ante dating stage. This is in contrast to classes, which are defined based on ex-post acceptance decisions. The following proposition shows that, for any any contiguous set of classes n through $n+k$, an equilibrium exists where all agents in each of these classes pool on a single report $\hat{q}$. That is, any pooling structure that nests contiguous sets of classes within pools can be supported in equilibrium. Figure 3.2 shows an example with two classes pooling on a single report: all agents in classes

1 and 2 make the same report $\hat{q}$. Note that the cutoffs for these classes are endogenous in the reporting structure.

Proposition 2. *Any contiguous pool* $(q_j(n+k), q_j(n-1)]$ *can be supported in a BPPE.*

Proof. Suppose that, for each side j, all agents in classes n through $n+k$ pool on a single report $\hat{q}_j$, no agent outside $(q_j(n+k), q_j(n-1)]$ reports $\hat{q}_j$ and every agent rejects any report made only by agents outside of their class. Then if a pooling agent makes any report other than $\hat{q}_j$ they will never receive a match, yielding a non-positive payoff. Thus there is no profitable deviation for pooling agents. If each other class i forms a pool where all agents report $\hat{q}_{ij}$, this is an equilibrium, so a BPPE exists. □

We will now provide a lemma that establishes the range of characteristics pools can have in equilibrium. In particular, it shows that pool cutoffs can coincide with class cutoffs as above or appear within classes, with the possibility of multiple pools within a class and mutual rejection by agents within the same class but in different pools, even though they would like to match ex-post.

Lemma 5. *For any contiguous pool in a BPPE, on at least one side of the market, one of the following holds for the lower(upper) bound of the pool:*

i) The lower(upper) bound is also the lower(upper) cutoff of the lowest(highest) class with reports in the pool.

ii) the non-endogenous pool cutoff induces indifference between reporting within the pool and giving any report given by agents in the lowest(highest) pooling class but outside the pool.

Proof. Without loss of generality, consider the lower bound case. Suppose i) and ii) are violated and the pool is consistent with equilibrium. Then there is a report made in equilibrium by an agent in the lowest pooling class with a different payoff than some other report made by an agent outside pool but in the class. Then agents in that class have a strict incentive to give the higher payoff report and the assumed reporting is not optimal. Contradiction. □

The above lemma could be presented more tersely by giving a more general form of ii., but this formulation provides more intuition about the range of possible reporting strategies. It suggests several different sorts of equilibria, and we will describe examples for both cases. If reports satisfy i) for both bounds, classes are nested within pools and agents can't profitably leave the pool by reporting above, where they'll get rejected, or reporting below, where they'll get a lower payoff. If reports satisfy ii) for both bounds, agents will reject one another ex-ante even though they are in the same class. For example, suppose a report $\hat{q}_m$ is made by some men in class n and some outside of it, and $\hat{q}_w$ is made by some women in class n and some outside. If men reporting $\hat{q}_m$ always reject women reporting $\hat{q}_w$ and vice versa, deviating to accepting may yield a costly date with no match and will never yield a match, so it is strictly better to reject. This can be supported if the payoff for each report on a given side in class n is equal, e.g. half of n men of each type in the class report $\hat{q}_{m1}$ and only accept $\hat{q}_{w1}$ and the other half report $\hat{q}_{m2}$ and only accept $\hat{q}_{w2}$, and women behave symmetrically. Note that reports need not satisfy the same case for both the top and the bottom bounds– the upper bound could satisfy i) while the lower bound satisfies ii)

This can analysis can trivially be extended to pools that are discontiguous but can be represented as a finite union of contiguous pools. In fact, we can quite easily find equilibria where, for example, classes 1 and 3 pool on a single report despite rejecting one another ex-post while class 2 agents make a report that uniquely identifies their class.

Corollary 1. *Suppose the support of a pool can be expressed as the union of a finite set of intervals. Then each interval must satisfy Lemma 5.*

Thus we see that the structure of reporting can be highly non-monotonic, and agents within a given class can even reject each other ex-ante. However, the underlying ex-post matching structure retains the coarsely assortative class structure found in previous research in this environment with observable types.

3.2.3. *The Limited Pooling Equilibrium.* Having established some characteristics of the set of possible equilibria, we will now introduce the equilibrium[21] of interest for the remainder of this paper. Unlike many other equilibria in this environment, this equilibrium looks very similar to those found in Macnamara and Collins (1990) and Burdett and Coles (1997), with agents only dating within their class. The primary difference from equilibria with observable types is that there is a region between the ex-ante and ex-post (class) cutoffs for each class that, with observable types, would be rejected by the agents in the class. However, since per-date costs induce a lower ex-post cutoff, agents in this interval can pool with those above them and get accepted due to the laxer ex-post standards.

Definition 4. *(Limited Pooling Partial Equilibrium (LPPE))* We'll call a BPPE a*n LPPE* if each agent makes a report only made by agents in their class and accepts every report made by agents in their class.[22]

We can now easily characterize the equilibrium pooling structure and the relationship between the ex-ante and ex-post acceptance decisions:

Lemma 6. *(1) In any LPPE, agents between the ex-ante and ex-post cutoffs must pool with agents above the ex-ante cutoff such that the expected quality of that report exceeds the ex-ante cutoff.*

(2) In any LPPE, agents will always accept after a date.

Proof. Consider an agent $m(w)$ in class n. Suppose a report $\hat{q}$ is never made by women(men) above the ex-ante cutoff in class n. Then $m(w)$'s continuation value is higher than the expected payoff of dating and matching to the $\hat{q}$ woman(man) and he must reject. Thus women(men) below the ex-ante cutoff must pool with agents above it to gain acceptance in their class. It is immediate that any agent with a type below the ex-ante cutoff must pool with an agent of a type above the ex-ante cutoff for the average quality of an agent in that

[21]Formally, a set of equilibria with reporting strategies that lead to equivalent payoffs for all agents.

[22]Note that there is a larger set of equivalent equilibria where every agent accepts every agent in their class ex-ante and only goes on dates with agents in their class, but where, for example, men make informative reports and accept all matches and women make uninformative reports and are selective. It is without loss to consider the special case of Definition 4.

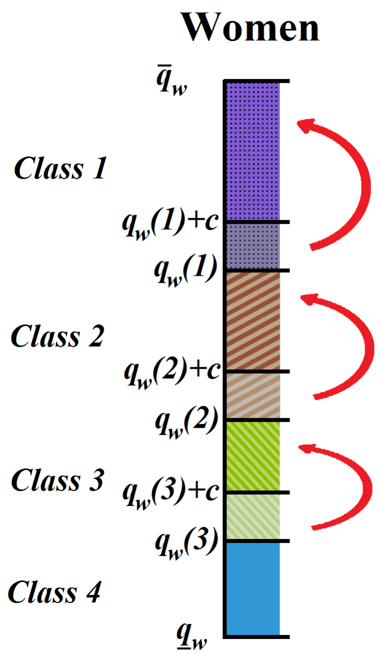

FIGURE 3.3. The pooling structure of an LPPE.

pool to exceed the ex-ante cutoff. Finally, since agents only accept dates within their class, they always accept a match after a date. □

Figure 3.3 shows the pooling structure of an LPPE. For women(men) in each class, agents the region from $q_w(n)$+c to $q_w(n)$ are below the ex-ante class cutoff but above the ex-post class cutoff, and thus will be rejected ex-ante if they reveal their true type, but will be accepted ex-post. Thus, they must pool with agents above the ex-ante cutoff (classes will always have length greater than c, so this is always possible). Note that, in this example, $\underline{q}_w > q_w(4) + c$, so the fourth class ends before the ex-ante cutoff. While the first J-1 classes must end with the class's endogenous ex-post cutoff, the last class J may end with the lower limit of the support of G, which may be above $q_w(J)$ or even above $q_w(J) + c$.

Lemma 6 shows that agents between the two cutoffs must pool in order to maximize their payoffs. Comparing an LPPE to the larger set of equilibria, we see two primary ways other equilibria differ: we can find equilibria where multiple classes pool reports and we have between-class date acceptance, and we can find equilibria where agents in a given class reject dates with others within that class due to coordination failures.

The lack of a complex reporting structure independent of ex-post classes make LPPE very tractable, and thus very attractive. There are reasons beyond convenience to focus on LPPE, though. Given linear returns to matching (LRM) and a steady-state assumption these equilibria must exhibit the finest class partition in the set of steady-state equilibria in terms of the number of classes.[23]

Additionally, in any other equilibrium some agents must pool and accept dates they will later reject, or reject dates they'd like to accept. This means that every such agent would strictly prefer to leave the pool and report their true class, and they are only prevented from doing so by the absence of messages that can reveal their class.[24] In particular, any man(woman) who goes on dates outside their class or rejects dates inside their class would prefer to unilaterally reveal their true class and commit to accepting agents in their class ex-post, since this would allow them to match to every good draw and to reject every bad date. This does not imply a simple equilibrium dominance argument, unfortunately–everyone switching to truthful reporting of class creates higher expected payoffs, which induces higher class cutoffs. Thus, agents between the old and new cutoffs could be made worse off by being bumped from class n to class $n+1$.

However, given LRM, consider a non-LPPE equilibrium. There must be some class where men(women) reject dates within the class or accept dates outside the class. For the first such class, n, class n men(women) who would remain in the class under the limited pooling class cutoff would all strictly prefer to form a coalition and report their true class, if that were possible. Importantly, if they did, no man(woman) outside their class would have an incentive to re-pool with their new report, since they would simply be rejected ex-post, and any coalition attempting to repool would necessarily include agents who would be worse off under the coalitional deviation. Thus, non-LPPE seem fragile in the sense that all agents will deviate from the multi-class pools that define them if they can find a way to report their

[23]Depending on the parameterization, there may be other equilibria with the same number of classes, but none will have fewer.

[24]We'll proceed informally here. Formally modeling this game with coalitions, repeated reports, and deviations that induce off equilibrium path beliefs and play outside of steady-state would be intractable.

true type, and when agents deviate from these pools there is no incentive for the agents in their former pool to unilaterally follow them to a new report.

Could this same argument also exclude the LPPE, where pooling also occurs? We'll provide a heuristic argument to the contrary. In an LPPE, high type agents in a class would like to reveal their true types and thus escape pooling with agents between the ex-ante and ex-post cutoffs. However, all of these low type agents strictly prefer to pool with higher type agents in the class. If we assume that agents can change reports at intervals and coalitional deviations are significantly more costly than unilateral deviations due to coordination costs, we'd expect to generally see pooling as in the LPPE since it will be too costly for high types to repeatedly coordinate on new reports only to have low types unilaterally follow them. The finest partition and equilibrium preference arguments are formalized in Section 3.3.1.

We can now write down the explicit form of the agents optimization problem. We'll focus on the case where the distributions of men and women and their utility functions are symmetric for tractability. Define the number of classes as J, the proportion of agents in class n $\lambda_n \equiv G(q(n-1)) - G(q(n))$, and define q_l and q_u as the lower and upper cutoffs for a class, respectively. Given that agents accept any agent in their class and reject all others ex-ante, every date results in a match and the probability of accepting a draw is λ_n. Then equation 3.3 can be rewritten as

$$q_l = \frac{\alpha\lambda}{\lambda\alpha + r}\int_{q_l}^{q_u}(x-c)\frac{g(x)}{\lambda}dx \tag{3.4}$$

Rearranging and applying integration by parts, we have:

$$q_l = \frac{\alpha}{r}(\int_{q_l}^{q_u} G(q_u) - G(x)dx - \lambda c) \tag{3.5}$$

We can now characterize the class structure explicitly:

Proposition 3. *Given G, a LPPE implies sequence of cutoffs for men and women* $\{q(n)\}_{n=0}^{J}$ *satisfying* $q(n) = \frac{\alpha}{r}(\int_{q(n)}^{q(n-1)} G(q(n-1)) - G(x)dx - \lambda_n c)$, where $q(0) = \overline{q}$, *and* $q(J) <= \underline{q}$.

Proof. The first and third claims follow directly from Proposition 1 and equation 3.5, and if the fourth were not true there would be another class $J+1$. □

It also follows that agents accepting measure zero masses of agents outside their class or rejecting measure zero masses of agents inside their class generates the same cutoffs and payoffs.

3.3. **Steady State in the Limited Pooling Equilibrium.**

3.3.1. *Linear Returns to Matching.* The linear returns to matching equilibrium analysis closely follows Burdett and Coles (1997). Some proofs go through nearly unchanged, but others must be amended to account for per-date costs and the differing assumptions on returns to matching. Define the distribution of agents leaving the platform as $H(q)$ and the mass of agents leaving the platform by O. We can now define our complete equilibrium concept by combining the partial equilibrium of the LPPE, which ensures all behavior and beliefs are rational and assumes steady-state, with a balanced flow condition that ensures steady-state holds by equating the endogenous outflows with the exogenous inflows, closing the model.

Definition 5. (LPPE Steady-State Equilibrium (LSSE)): given exogenous inflows (F), a steady state equilibrium is pair (G, N) satisfying LPPE and balanced flow: for every interval $[q_1, q_2) \in [\underline{q}, \overline{q}]$, $O(H(q_2) - H(q_1)) = F(q_2) - F(q_1)$.

The cutoff equation is now

$$q(n) = \frac{N}{r}\left(\int_{q(n)}^{q(n-1)} G(q(n-1)) - G(x)dx - \lambda_n c\right) \tag{3.6}$$

Within a given class, we can get a simple characterization of outflow. Outflow in a class is given by the number of agents on the platform, N, times the proportion of agents in the class, λ_n, times the rate of draws of an agent in that class $N\lambda_n$. Then outflow from class n is $\lambda_n^2 N^2$. Then, in an LSSE,

$$\lambda_n = \sqrt{(F(q(n-1)) - F(q(n)))}/N \tag{3.7}$$

We also have that, for any $[q_1, q_2)$ in class n, $\lambda_n(G(q_2) - G(q_1))N^2 = F(q_2) - F(q_1)$ and thus, with the differentiability of F,

$$g(q) = \frac{f(q)}{\lambda_n N^2} \tag{3.8}$$

Thus the density of agents on the platform in a given class is inflow density times a scalar. Combining (3.6) and balanced flow, we can get eliminate G terms, yielding class cutoffs solely in terms of inflows and c.

$$q(n) = \frac{1}{r}\Big(\int_{q(n)}^{q(n-1)} \frac{F(q(n-1)) - F(x)}{F(q(n-1)) - F(q(n))} dx - c\Big)\sqrt{(F(q(n-1)) - F(q(n)))} \tag{3.9}$$

Note that in the linear returns environment, the N's cancel out, and we have cutoffs that depend on the previous cutoff. We can now explicitly characterize the LSSE in this environment:

Proposition 4. *Given F, then (G, N) defines a LSSE if and only if G satisfies (3.8) and λ_n satisfies (3.6), (3.7), $q(0) = \overline{q}$, $q(J) \leq \underline{q}$, and $\sum_n \lambda_n = 1$.*

Proof. $\sum_n \lambda_n = 1$, the boundary conditions, and (3.6)-(3.8) are necessary in an LSSE by construction. Conversely, the assumptions guarantee $G(\overline{q}) = 1$, $G(\underline{q}) = 0$ and G increasing, so G is a well defined steady state distribution and any G and N satisfying them form a valid LSSE. $\square$

Thus, equilibrium requires that each class cutoff is the solution to the agents' optimal stopping problem and the density on the platform is consistent with balanced flow. $\sum_n \lambda_n = 1$ ensures that g is well defined.

To ensure existence of an LSSE, we'll need to make some distributional assumptions. An increasing hazard function will ensure that the class structure is unique. Note that, while Burdett and Coles need this assumption to deal with a multiplicity of cutoffs due to N, that channel is shut down in the linear returns environment. However, this assumption also constrains multiplicity induced by per-date costs, so it is still necessary in this environment.

- **Assumption 3 (HAZ)** : The hazard function $f(q)/(1-F(q))$ is increasing in q.

Proposition 5. *Given F, the partition satisfying (3.6)-(3.8) and the boundary conditions is unique.*

Proof. See Appendix 6.2.1. □

The intuition for this proposition is that the RHS of (3.9) is decreasing in $q(n)$, while the LHS $q(n)$ is obviously increasing, yielding a single crossing. The RHS can be interpreted as the expected surplus quality of an accepted match over the cutoff quality, multiplied by the probability of acceptance. Generally, we'd expect this to be decreasing in cutoff type $q(n)$, since a higher cutoff lowers the surplus over cutoff for any given draw, and, were G exogenous, a higher cutoff would lower the probability of accepting a draw. However, due to the endogenous nature of G, it's possible for the density to rise as $q(n)$ increases, swamping the aforementioned effects. The hazard rate assumption ensures that the density can't rise too fast, excluding this possibility.

In this environment, existence and uniqueness of the equilibrium follow directly. The class cutoffs are unique and these cutoffs imply a unique steady-state mass on the platform, N. This in turn ensures a unique density $g(q) = \frac{f(q)}{\sqrt{(F(q(n-1))-F(q(n)))N}}$ and thus a unique distribution on the platform G.

Proposition 6. *A unique LSSE exists.*

Proof. The class summation condition and (3.7) yield

$$N = \sum_{n} \sqrt{(F(q(n-1)) - F(q(n)))} \tag{3.10}$$

and N does not enter the cutoff equation so $q(n)$ is not a function of N and uniqueness is ensured. Existence is similarly direct. $\square$

We can also show that the cutoffs are decreasing in c–that is, increasing per-date costs generally makes classes coarser, and for sufficiently high c the class structure completely unravels as every man(woman) accepts every woman(man) due to the high costs of continuing their search and paying the per-date cost again.

Proposition 7. *Given LRM, $q(n)$ is decreasing and continuous in c in an LSSE for all n and $q(n) \to 0$ for c sufficiently high.*

Proof. Suppose c increases. Consider the first endogenous cutoff, $q(1)$. The RHS of (3.9) is decreasing in c, and the RHS is decreasing in $q(1)$ by Lemma 11, so the lower RHS from c must be compensated with a lower $q(1)$ for equality to hold. We can now proceed inductively. Suppose $q(n-1)$ is decreasing in c. Lemma 11 also shows that the RHS increasing in $q(n-1)$, so to maintain equality, $q(n)$ must increase in $q(n-1)$. Additionally the argument in the base case ensures that, fixing $q(n-1)$, $q(n)$ is decreasing in c. Thus, $q(n)$ is increasing in the argument that decreases, $q(n-1)$, and decreasing in the argument that increases, c, and must decrease on net. Direct inspection shows continuity given the continuity of F. Lemma 11 shows that, for sufficiently high per-date costs, the RHS goes to zero and thus the cutoff goes to zero. $\square$

The intuition for this result is that per-date costs lower expected match utility, and in the optimal stopping problem cutoff utility must be equal to expected match utility, so higher per-date costs should yield lower cutoffs.

We're now ready to formalize the justifications for our equilibrium selection. Lemma 7 shows that LPPE classes with the same upper bound generate higher payoffs for agents within the class and thus that the classes are smaller. Lemma 8 shows that the top agents in a pool prefer revealing their class and matching to one another to remaining in the pool. Corollary 2 shows that an LPPE induces the finest partition of the type-space in terms of classes. Recall that $q(n-1)$ is the upper bound of a class n and define $q(LPPE, q(n-1))$ as the cutoff induced by $q(n-1)$ if everyone in the class accepts one another ex-ante and no one else, as in an LPPE and n_{LPPE} as the corresponding class starting at $q(n-1)$ with no cross-class pooling or within-class rejection and where balanced flow is satisfied. Also define $n_{LPPE,q(n)}$ as the class with upper bound $q(n-1)$, lower bound $q(n)$, limited pooling and balanced flow and define $q(LPPE, n)$ as the nth cutoff given an LPPE. Define M_n as the mass of agents in class n.

Lemma 7. *Given LRM, a class n starting at $q(n-1)$ where the probability of an agent in n accepting a date outside the class or rejecting a date inside the class is strictly positive must have a cutoff $q(n) < q(LPPE, q(n-1))$. Additionally, $q(n) < q(LPPE, n)$.*

Proof. See Appendix 6.2.1. □

Lemma 8. *Given LRM, consider an equilibrium where the probability of an agent in a class n accepting a date outside the class or rejecting a date inside the class is strictly positive. Further, define n as the first class where this is true. Then agents in class n above $q(LPPE, q(n-1))$ all strictly prefer an equilibrium with class n_{LPPE} to the one with class n, and if they could coordinate to reveal their true class and accept only others in their class, no agent outside their class would have an incentive to pool with them.*

Proof. A class with a lower cutoff of $q(n)$ must have an expected match quality $q_E(n) = q(n)$ and one with $q(LPPE, q(n-1))$ must have $q_E(n_{LPPE}) = q(LPPE, q(n-1))$. By Lemma 7, $q(n) < q(LPPE, q(n-1))$, so agents must prefer the n_{LPPE}. Additionally, any agent outside the class that pools with them will be rejected ex-post, and thus has no incentive to pool. □

Corollary 2. *In any LSSE with LRM, the nth class cutoff is maximal in the set of steady-state equilibria, and the number of classes is also maximal.*

Proof. By Lemma 7, $q(n) < q(LPPE, n)$ and if $q(k)$ exceeds the lower bound of the distribution, $q(LPPE, k)$ must as well. □

4. Strategic Platforms

4.1. Per-Date Costs as Frictions.

4.1.1. *Monopolist Platform.* Up until now, we've taken per-date costs as given, but a strategic platform such as a social planner or profit maximizing monopolist may be able to influence them, either by increasing them via per-date pricing, or decreasing them by providing easy ways for agents to communicate and verify type (e.g. video chat) or verifying certain aspects of an agent's report. We'll first consider the frictional case with a monopolist platform. Specifically, we'll consider a platform that charges a fixed fee for both sides of the market, can provide its service costlessly, faces an exogenous per-date friction $\bar{c}$, and can decrease the per-date cost to c at a cost $\tau(c)$, where $\tau(\bar{c}) = 0$ and τ is strictly decreasing in c. Thus the firm will charge a fixed fee p and every agent above a cutoff $q(p, c)$ will join the platform, yielding profit flow rate $\Pi_{fric}(p, c) \equiv p(F(\bar{q}) - F(q(p, c))) - \tau(c)$, and p will equal the expected payoff of the lowest joining agent. Formally, amend the game to include a first stage where the platform chooses p and c and consider the equilibrium where the maximal mass of agents join the platform. As before, agents have an outside option of zero[25]. Define $q_n(c)$ as the nth endogenous cutoff given c. Define $p^*(c)$ as the optimal price given a per-date cost c. Then it will generally be optimal for the platform to set a price such that the lowest type joining the platform is also the cutoff type for the last class joining the platform. Specifically,

Lemma 9. *Suppose LRM and $\bar{c}$ sufficiently high. Either:*

[25]This can be relaxed throughout Section 4. For example, the outside option can easily be re-specified as a time discounted random draw from the distribution of agents off the platform (large platform) or the overall distribution of agents in the market (small platform). Assuming that the search technology (draw rate) off platform is sufficiently slow, agents will optimally accept any draw off platform, rationalizing these specifications. All the results of Section 4 go through with these endogenous outside options, though the firm's optimal prices will change. If the search technology off platform isn't slow, high types may have better outside options than low types, limiting the extent to which a platform can profitably raise per-date prices or allow per-date frictions. Generally, the greater the efficiency advantage of the platform relative to the outside option, the more flexibility the platform will have to support high per-date costs and manipulate user behavior.

i) a positive c is optimal for the monopolist platform, or

ii) choosing a price that yields a cutoff $q(p,c)$ that does not coincide with the lowest joining class' endogenous cutoff is suboptimal for the platform if $q(p,c) < q_1(0)$.

Proof. Suppose the optimal contract yields a cutoff $q(p,c)$ such that $q(p,c) > q_1(c)$ and $q(p,c) < q_1(0)$. Then a positive c is optimal. Now suppose the platform induces an individual rationality (IR) cutoff $q(p,c) < q_1(c)$, and $q(p,c) \neq q_n(c)$ for any n. For sufficiently high $\bar{c}$, there must be a c' such that $\bar{c} \geq c' > c$ and $q(p,c) = q_1(c')$ since q_1 is continuous, decreasing in c, and goes to zero as c increases. Note that the cutoff type is equal to discounted expected utility. Thus, any class with the same lower cutoff yields the same expected match quality for agents in that class. Then if the firm chooses c' and a p' to induce the same cutoff, the quantity of agents on the platform is identical, but the cutoff agent is willing to pay $\phi(q(p,c))\psi(q(p,c))$, while under the original regime the cutoff agent is willing to pay $\phi(q_n(c))\psi(q(p,c))$. $q(p,c) > q_n(c)$ implies $p' > p$, and since cutoffs are decreasing in c, $\tau(c') < \tau(c)$. Thus the firm will increase profit by inducing $q(p',c') = q_1(c')$. □

We can use this to result to show that the platform generally will not have an incentive to lower per-date costs to zero:

Proposition 8. *Suppose LRM. If $\bar{c}$ is sufficiently high and $q(p^*(0),0) < q_1(0)$, the monopolist platform never has an incentive to lower per-date costs to zero.*

Proof. $q(p^*(0),0) < q_1(0)$, so since $q(1,c) \to 0$ as c increases and q(1,c) is continuous in c, the intermediate value theorem (IVT) ensures that there will be a $c^* > 0$ such that $q(1,c^*) = q(p^*(0),0)$, $p^*(c^*) \geq p^*(0)$, and $\tau(c^*) < \tau(0)$. Thus profit will be higher with a per-date cost c^* than with a zero per-date cost. □

Thus, even when per-date costs are frictions, a monopolist platform can profitably withhold higher efficiency search technologies, even when the cost of implementing such technologies is minimal. Lemma 9 shows that inducing endogenous class cutoffs equal to the IR cutoff is typically optimal, since that is the highest class cutoff that indifferent agent can have and the higher a cutoff is, the higher the utility agents in the class receive due to the equality of the cutoff and the continuation value. Then, given that prices and per-date costs are chosen to induce this coincidence between the platform and last class cutoff, higher per date costs have a direct effect of decreasing total surplus via effort spent on dates, but at the same time transfer surplus from high types to low types. Generally, the net effect of these countervailing forces would be ambiguous, but because the cutoff is held constant they must cancel out exactly, again due to the equality between cutoffs and continuation values. Thus, in terms of revenue, the platform is indifferent between any per-date costs that induce the appropriate cutoff, and strictly prefers higher per-date costs in terms of its own cost τ.

4.1.2. *Social Planner.* We can now consider analogous social planner's problem. Here, it is much less likely that it will be efficient to have positive per-date frictions. However, as was mentioned before and will be elaborated in the next section, there are externalities that can make agents too picky, and this can lead to inefficiently small classes and large utility losses due to discounting–agents spend too much time searching due to other agents' pickiness, and end up worse off. Also, the last class is qualitatively different then the preceding J-1 classes–it may not end at the endogenous cutoff, but rather at the bottom of the support of the distribution, which may be above the endogenous cutoff. This can lead to tiny rump classes with very low payoffs since there are very few agents in the class and thus the average search time is extremely high. In addition to directly lowering payoffs through frictions, c can thus speed up matching and change the size of this last rump class, potentially avoiding inefficiently sized final classes. Note that the flow of total surplus can be expressed as

$$TS \equiv \sum_{n=1}^{J} q_{jE}(n) \int_{q(n)}^{q(n-1)} \psi(q) f(q) dq \tag{4.1}$$

That is, for each class n, the rate of total surplus generation is the expected match utility for each agent integrated over the distribution of inflow in class n, where expected match quality for the class $q_{jE}(n)$ is constant across agents and can be pulled out of the integral. Total surplus is the sum over these classes. For ease of exposition, suppose for now that $\psi(q) = 1$. As shown in Appendix 6.2.3, we can express $\frac{\partial TS}{\partial c}$ as the sum of the effects of the change in each class cutoff $\frac{\partial q(n)}{\partial c}$ on the surpluses generated in the classes above and below it. In particular, the net effect of a change in cutoff $q(n)$ is proportional to

$$F(q(n-1)) - F(q(n)) - f(q(n)) \cdot (q(n) - q(n+1)) \tag{4.2}$$

as shown in Figure 4.1. If $F(q(n-1)) - F(q(n)) > f(q(n)) \cdot (q(n) - q(n+1))$ for all n (with a caveat for the last class discussed in the Appendix), TS decreases in c, and if the inequality is reversed the opposite holds. While the condition itself is simple, the class cutoffs are determined by a highly non-homogeneous recurrence relation, so finding conditions for either case is quite difficult in general. Below, we treat the case when utility is highly supermodular, which ensures Equation 4.2 is positive because the trade-off is between utility for class n, $F(q(n-1)) - F(q(n))$, and utility for class $n+1$, $f(q(n)) \cdot (q(n) - q(n+1))$. Supermodularity ensures that higher classes generate more surplus since they are populated by higher type agents, so for sufficient supermodularity the effect on class n dominates the effect on class n-1.

Proposition 9. *Suppose LRM,$\psi(q) = q^\alpha$, and $\tau(c) = 0$. If α is sufficiently high, total surplus is decreasing in c.*

Proof. See Appendix 6.2.3. □

In the Appendix 6.2.3 we run numerical simulations for the modular utility case, and find that increasing c typically decreases total surplus, and must decrease it above a certain point (if c gets sufficiently high no one can get positive utility from joining the platform). The primary reason for this is that lower type classes tend to be smaller, since agents choose their reservation types based on a trade-off between quality and discounting, which causes a proportional decrease in match utility. Thus, agents who get high expected payoffs must be less selective, as waiting is more costly for them. Since $\frac{\partial TS}{\partial c}$ is negative when the surplus loss for higher classes of shifting a cutoff down outweighs the gain to lower classes, lower classes having less mass and thus generating less surplus makes $\frac{\partial TS}{\partial c} < 0$ likely. Equivalently, the direct effect of decreasing surplus due to frictions overwhelms any efficiency gains due to

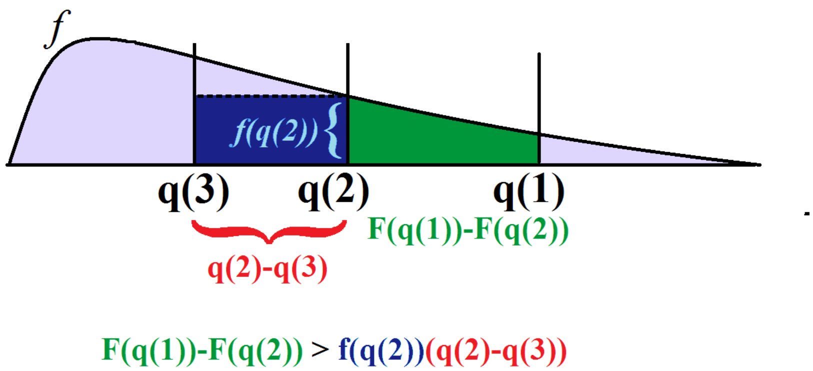

FIGURE 4.1. An example showing the trade-off induced by $q(2)$ shifting due to an increase in c. Class 2 surplus decreases proportionally to its mass (in green) due to the class cutoff $q(2)$ shifting down and thus lowering expected match quality, which is equal to the class cutoff. Lowering $q(2)$ also bumps top class 3 agents up to class 2. The surplus increase induced by this is proportional to the change in payoff $q(2) - q(3)$ (red) multiplied by the density of agents at the cutoff $f(q(2))$ (blue). Geometrically, this is the blue rectangle in Class 3. If the former decrease (green area) exceeds the latter increase (blue area) for each class, increasing c will lower total surplus.

lower selectivity. However, at some values of c $\frac{\partial TS}{\partial c}$ can be increasing, due to the effect of c on time discounting and the rump class. This is especially common with decreasing distributions that yield more mass in lower classes. Decreasing distributions also put more weight on the rump class, creating periodicity in the surplus as the class cutoffs shift downward in c and the rump class goes from being a large, relatively efficient class to a small class whose size is limited by the support of the distribution, and then to a large class again as the last class cutoff passes the bottom of the support of the distribution and the next class becomes the last class. Thus, for friction reduction costs where small decreases in frictions are asymptotically costless like $\tau(c) = (\bar{c} - c)^2$, we can find cases where there is no incentive to decrease per-date frictions even a little bit, since total surplus is locally increasing in c.

4.2. **Additively Separable Match Utility and Per-Date Prices.**

4.2.1. *Monopolist Platform.* We'll now consider the case of per-date costs as prices. We'll need to restrict our attention to the case when $\psi(q) = 1$ for tractability–platforms charging a per-date price proportional to agent type is inconsistent with the unobservable types assumption of this paper. Conversely, charging a fixed per-date price with supermodular match utility won't induce the class structure that makes this analysis tractable–higher type agents in any potential class will be less affected by the per-date cost than lower type agents in that class, and will thus have different cutoffs. We study numerical simulations for supermodular cases in Appendix **??**. First consider the case of a monopolist platform. The platform charges a fixed fee p and a per-date price c. Consider the equilibrium where the maximal mass of agents join the platform. Since $\psi(q) = 1$, social surplus is modular–the total surplus is just the sum of the match payoffs $\phi(q)$ for each side, discounted by the expected time to match. Thus the structure of matching has no effect on payoffs, only the speed of assignment matters. Of course, individual agents benefit from matching to high types, but their benefit comes at a cost to other agents who don't get high type matches. Thus, externalities generally induce agents to be selective when matching, even though a social planner would assign every agent on the first draw. Thus, increasing per-date prices improves social surplus. It is also clear that, if a monopolist prefers positive per-date frictions, they will prefer positive per-date prices as well, which is the basis on which Corollary 3 extends the frictional results of the last subsection to the per-date price environment. Given profits under frictions of $\Pi_{fric}(p, c) \equiv p(F(\overline{q}) - F(q(p, c))) - \tau(c)$, profits under prices are $\Pi_{price}(p, c) \equiv (p + w(p, c)c)(F(\overline{q}) - F(q(p, c)))$, where w is the average of the expected discounting for each class weighted by the inflow rate of the class.

Corollary 3. *Suppose LRM. If $q(p^*(0), 0) < q_1(0)$, the monopolist platform never has an incentive to lower per-date prices to zero.*

Proof. Note that the proofs of Lemma 9 and Proposition 8 go through with the removal of $-\tau(c)$ in the profit function and the inclusion of $w(p, c)c(F(\overline{q}) - F(q(p, c)))$. □

4.2.2. *Social Planner.* As discussed above, the social planner prefers agents to leave as quickly as possible to minimize time discounting, since assignment doesn't matter. We can prove this result very directly for constant returns to matching (CRM):

Proposition 10. *Suppose CRM. If c is a price, $\psi(q) = 1$, $f(x)$ is increasing or $xf(x)$ is increasing and c is sufficiently small, and there is more than one class when c is zero, a positive c maximizes social surplus.*

Proof. A sufficiently high c will ensure a single class, and we have multiple classes with zero per-date costs.$\psi(q) = 1$, so, given the inflow distribution F, social surplus is given by $SS = 2\int_{\underline{q}}^{\overline{q}} \phi(q)f(q)E[e^{-rt}|q]dq$. This is maximized when $E[e^{-rt}|q]$ is maximized for all q. A single class maximizes the exit rate, maximizing $E[e^{-rt}|q]$. Thus a single class maximizes social surplus and a positive c is necessary to induce a single class. Platforms can choose an appropriate fixed fee (possibly negative) to satisfy agent IR constraints. □

In the LRM case, it's not necessarily true that the platform will want to maximize outflow for every agent type. Since the mass on the platform determines the rate of draws, having more agents on the platform can boost exit rate and thus discounted utility. If the mass of all agents is higher, this is a second order effect that must be overwhelmed by increased agent selectiveness in order to induce larger masses on the platform in the first place, but it is possible that the planner would benefit from inducing low types to reject other low types and only match to high types. High types give a higher payoff to their match, so time discounting is more costly for them. By contrast, a sufficiently low type agent contributes almost nothing no total surplus. Thus, inducing low types to reject other low types and increase their mass on the platform would allow high types to get quick matches and preserve their much more valuable contribution to match surplus. In this environment, the platform has no means to induce this partially negatively assortative matching[26] so inducing a single class will still be optimal given the instruments available, but we'll need to amend the proof to take into account the endogenous draw rate.

[26]Negatively assortative matching occurs when higher types match to lower types rather than their own type.

Proposition 11. *If c is a price, $\psi(q) = 1$, LRM holds, and there is more than one class when c is zero, a positive c maximizes social surplus.*

Proof. A sufficiently high c will ensure a single class, and we have multiple classes with zero per-date costs.$\psi(q) = 1$, so, given the inflow distribution F, mass on the platform will be 1 by (3.10) and every agent will leave upon a draw, which they get at rate 1. Suppose there is more than one class. Mass on the platform is $\sum_n \sqrt{(F(q(n-1)) - F(q(n)))}$ by (3.10), and the probability an agent in class n gets a draw they will accept is $\frac{\sqrt{(F(q(n-1))-F(q(n)))}}{\sum_n \sqrt{(F(q(n-1))-F(q(n)))}}$. Then the rate of accepted draws is $\sqrt{(F(q(n-1)) - F(q(n)))} < 1$ and all agents have a longer expected wait on the platform, getting lower utility in expectation. Platforms can choose an appropriate fixed fee (possibly negative) to satisfy agent IR constraints. □

5. Conclusions

In this paper, we extended the NTU search literature to an environment with a cheap talk stage and costly type verification. We found that the partition or class based equilibria that have characterized this literature extend to this environment with informational frictions, with agents only matching to one another within their respective disjoint classes. When per-date costs are endogenously chosen by a strategic platform, positive per-date costs may be optimal, despite being distortionary, and, in the case of per-date frictions, having a negative direct effect on surplus. A social planner can take advantage of these per-date costs by using them to counter externalities that make agents too picky by allowing low type agents to pool with high type agents, preventing those high type agents from inefficiently rejecting them. A monopolist can use per-date costs to induce effective transfers from high to low types by forcing high types to match to low types, flattening the demand curve and allowing a monopolist that charges a fixed fee to extract more surplus from consumers.

Future avenues for study include analysis of more complex contracts in this environment to see how much profit the simple contracts commonly in use leave on the table, to see how more complex contracts interact with the externalities in these markets, and to more precisely capture the second degree price discrimination common to the menu of contracts in these markets. This is briefly studied in Appendix 6.1.1.

Studying an analogous model with transferable utility would be extremely useful, yielding results more applicable to job search, where cheap talk on both sides of the market can also be important. A two-type TU model is studied in Appendix 6.1.2, and we discuss how, under TU with linear returns to matching, agents are generally not picky enough since spending more time on the platform increases the mass of agents and thus increases the frequency of draws, a benefit agents do not internalize even with transfers known as the thick market externality. This is in contrast to the NTU case where agents are too picky, and thus yields opposite implications for per-date pricing, making negative per-date prices that incentivize an agent to stay on the platform longer optimal.

Including competition would be an obvious extension of this research program, though an even greater multiplicity of equilibria must be contended with due to the coordination issues with multiple platforms and network effects. Including exogenous exit and match dissolution would allow for more realistic modeling of matching behavior, especially on platforms that focus on short term matching like Tinder. This is not likely to substantively change the qualitative characteristics of the equilibrium, though.

6. Appendix

6.1. **Extensions.**

6.1.1. *More Complex Contracts.* So far we've assumed a simple contract structure with a single fixed fee and per-date cost motivated by the observed simplicity of contracts in this market. However, in many cases this is not optimal. With frictions, it will generally be optimal for the monopolist to offer different contracts based on report. We will focus on a menu of fixed fees with a constant per-date cost. Optimal menus of per-date costs will be highly dependent on the distribution F, and aren't amenable to a simple analysis.

Given the IR cutoff induced by a single fixed fee contract, if it's possible to include multiple classes above that cutoff, the original IR cutoff can be maintained with additional higher price cutoffs for higher types, allowing more surplus extraction. Under modular utility it's optimal to set contract intervals coinciding with endogenous class cutoffs (everyone reporting in class n must pay p_n) for reasons analogous to those in Lemma 9. If utility is supermodular it could be possible to find cases where this is not true, due to higher utility for higher types, but we'll assume one and only one contract per class, noting that, for supermodular utility, this must be weakly worse than the optimal contract structure.

For the lowest class k, the IR binds as is standard, yielding $p_k = q(k)\psi(q(k))$ For class $k-1$, note that lower class agents cannot deviate to higher classes due to rejection, even if the contract is more favorable. Thus we only need to worry about deviations by higher type agents to lower reports. The incentive compatability (IC) constraint $IC_{k-1,k}$ requires $q(k-1)\psi(q(k-1)) - p_{k-1} \geq q(k)\psi(q(k-1)) - p_k$, based on the bottom agent $k-1$[27] so $(q(k-1) - q(k))\psi(q(k-1)) \geq p_{k-1} - p_k$ and so on, with each following price p_{n-1} rising based on the difference in class utilities $q(n-1) - q(n)$ scaled by the supermodular component $\psi(q(n-1))$ and with rents $q(n-1)(\psi(q) - \psi(q(n-1)))$ accruing to class $n-1$ agents based on ψ increasing in q over the class interval. This strategy maintains the same total surplus but allows the platform to extract more from users, and is a lower bound for maximal revenue with multiple contracts. Some rents are still left the to users if utility is supermodular.

In the modular case, however, the monopolist has full extraction and is essentially a social planner, and thus the earlier analysis of the social planner's problem in Section 4 applies: increasing per-date frictions is less costly in terms of revenue than the direct effect of frictions on utility would imply, just like the social planner case, and, generally, smaller τ's can rationalize high per-date costs than one woud expect based on the direct effect of frictions (in particular, a sufficient τ would be $\tau(c) = \bar{c} - c$), especially under certain distributional assumptions as discussed in the frictional social planner case. Generally, however, positive per-date frictions are undesirable. With per-date prices and modular utility, a single fixed

[27]Higher type agents in the class benefit weakly more from higher quality matches, but pay the same price, so if the IC is satisfied for the lowest agent in must be satisfied for all others. This also makes showing local IC sufficient.

fee is optimal since it's optimal for the social planner. With supermodular it may still be optimal, especially for low degrees of supermodularity, but it may not be.

6.1.2. *Transferable Utility.* While we don' treat a transferable utility model in the main body of this paper, it is of significant interest, since it may be more applicable to many job market applications, and some prefer the TU assumption in models of dating and marriage. In this environment, externalities resulting from socially inefficient acceptences and rejections are eliminated by transfers as in the Coase theorem, but externalities resulting from the effects acceptance and rejection have the on mass of agents on the platform and their distribution persist. Under LRM, there is no cost to having too many agents one does not want to match to on the platform (the congestion externality), but there is a cost to having fewer agents one does want to match to on the platform (the thick market externality). Thus, there is only one externality in play: staying on the platform longer benefits agents who would like to match to you and has no effect on agents who don't, so agents aren't picky enough because they don't interalize the benefits their presence has for others. This is the opposite of the net effect of externalities in the NTU case, and suggests that platforms ought to lower per-date costs as much as is feasible.

We'll illustrate this by studying a TU version of this paper's model. Unfortunately, transferable utility greatly complicates the analysis by making match payoffs contingent not just on agent types but also on the endogenous outside options of each agent. However, we can analyze a two-type analogue of the model, with high types h and low types l and symmetric distributions. We'll focus on the match surplus function $u(h,h) \equiv 1$, $u(h,l) \equiv \beta$, $u(l,l) \equiv \gamma$, $1 > \beta > \gamma$. We'll say u has (weakly) supermodular payoffs if $2\beta \geq 1 + \gamma$ and assume this for the remainder of the section. Suppose an inflow rate normalized to 1, with the inflow of h types f, and the proportion of h types on the platform g and a mass of agents on the platform N. Suppose per-date costs are zero. When high types only accept high types, $gN = \sqrt{f}$, $(1-g)N = \sqrt{1-f}$, $g = \frac{\sqrt{f}}{\sqrt{f}+\sqrt{1-f}}$. Expected discount is then $\frac{gN}{gN+r} = \frac{\sqrt{f}}{\sqrt{f}+r}$ for high types and $\frac{\sqrt{1-f}}{\sqrt{1-f}+r}$ for low types. Then, given inflow rates f and $1-f$, the rate of surplus generated by a match is $f\frac{\sqrt{f}}{\sqrt{f}+r}1$ for the high type and $(1-f)\frac{\sqrt{1-f}}{\sqrt{1-f}+r}\gamma$ for the low type. When all agents accept one another, $g = f$ and $N = 1$. Expected discount is then $\frac{1}{1+r}$ for all types and the rate of surplus generated is $f\frac{1}{1+r}(f + (1-f)\beta)$ for high types and $(1-f)\frac{1}{1+r}(f\beta + (1-f)\gamma)$ for low types. Thus, separation is optimal if and only if $(1-f)\frac{1}{1+r}(f\beta + (1-f)\gamma) + f\frac{1}{1+r}(f + (1-f)\beta) \leq (1-f)\frac{\sqrt{1-f}}{\sqrt{1-f}+r}\gamma + f\frac{\sqrt{f}}{\sqrt{f}+r}$.

We can now study the TU equilibrium assuming Nash Bargaining. Then, given continuations values C_h and C_l, match payoffs after transfers are $u_h(h,l) = 1/2(\beta + C_h - C_l)$, $u_l(h,l) = 1/2(\beta + C_l - C_h)$, $u_l(l,l) = \gamma/2$, $u_h(h,h) = 1/2$. Suppose high types reject all low types. Then a high type receives expected payoff and continuation value $\frac{\sqrt{f}}{2(\sqrt{f}+r)}$, and a low type receives $\frac{\gamma\sqrt{1-f}}{2(\sqrt{1-f}+r)}$ and deviation to accepting a low type yields $(\beta + \frac{\sqrt{f}}{2(\sqrt{f}+r)} - \frac{\gamma\sqrt{1-f}}{2(\sqrt{1-f}+r)})/2$. Then separation isn't supportable in equilibrium when

$$\frac{\sqrt{f}}{2(\sqrt{f}+r)} < (\beta + \frac{\sqrt{f}}{2(\sqrt{f}+r)} - \gamma\frac{\sqrt{1-f}}{2(\sqrt{1-f}+r)})/2 \tag{6.1}$$

However, the social planner cares about the changed utility of the low type agent who matches to high type. Thus, the surplus for the two agents that match when the high type deviates is $\frac{\sqrt{f}}{2(\sqrt{f}+r)} + \frac{\gamma\sqrt{1-f}}{2(\sqrt{1-f}+r)}$ without the deviation and β when deviating. Then surplus is increased by deviating if and only if $\frac{\sqrt{f}}{2(\sqrt{f}+r)} + \frac{\gamma\sqrt{1-f}}{2(\sqrt{1-f}+r)} < \beta$, equivalent to the high type's inequality (6.1) . If the distribution on the platform was exogenous, this would conclude the analysis and the TU equilibrium would maximize total surplus. However, the mass of agents on the platform shrinks as agents become less picky, so a high type accepting low types imposes costs on others, Computing g given that a proportion x of high types accept low types and taking the limit as $x \to 0$, we find that a small mass x of h types deviating to accepting all lowers expected discount by $x\frac{(\sqrt{f}r)}{2\left(\sqrt{1-f}+r\right)^2}$ for high types and $x\frac{\left(\sqrt{1-f}r\right)}{2\left(\sqrt{f}+r\right)^2}$ for low types by decreasing the rate of draws. Then a high type agent accepting low types cannot be socially efficient unless

(6.2)
$$\frac{\sqrt{f}}{2(\sqrt{f}+r)} < (\beta + \frac{\sqrt{f}}{2(\sqrt{f}+r)} - \gamma\frac{\sqrt{1-f}}{2(\sqrt{1-f}+r)})/2 + \frac{r\left(-f^2\left(\sqrt{f}+r\right) - (f-1)^2 g\left(\sqrt{1-f}+r\right)\right)}{4\left(\sqrt{1-f}+r\right)^2\left(\sqrt{f}+r\right)^2}$$

where $\frac{r\left(-f^2\left(\sqrt{f}+r\right)-(f-1)^2 g\left(\sqrt{1-f}+r\right)\right)}{4\left(\sqrt{1-f}+r\right)^2\left(\sqrt{f}+r\right)^2} < 0$. Thus there is an interval where, under TU, high types will accept low types despite it being socially inefficient for them to do so–that is, agents are not picky enough.

6.1.3. *Non-Multiplicatively Separable Utility.* Multiplicative separability of the own-type component utility is a strong assumption in this paper. With modular utility and a constant per-date cost it is automatically satisfied since $\psi = 1$, but with supermodular utility it imposes a functional form restriction on match surplus and requires that per-date costs be a constant multiplied by ψ, meaning per-date costs must be higher for higher types and imposing a very strong relationship between match utility and per-date costs. This precludes constant per-date costs, making analysis of per-date prices with supermodular utility infeasible. Thus, we'd like to be able to say that this assumption, while necessary for tractability, is not driving our results. To assess this, we study a discretized analogue to our model, with five types ($q = .2, q = .4, q = .6$, $q = .8, , q = 1$) rather than a continuum. We need to limit the number of types because, without multiplicative separability, different agents in any candidate class will have different optimization problems and employ different cutoff strategies, precluding the discrete class structure that made analysis tractable. Without this, we'll instead find LSSE equilibria by brute force, testing every possible combination of cutoff strategies for each type for profitable deviations.[28] We'll also find optimal platform strategies as in Section 4 by testing every viable firm strategy (where price is the IR of the lowest joining type) and selecting the one that maximizes profit. We'll study the case where $\psi(q) = q^\alpha$ and per-date costs are constant. This will also allow us to look at per-date pricing when utility is supermodular. We'll use 3 different distributions, a decreasing distribution $(.35, .3, .2, .1, .05)$, the discrete uniform $(.2, .2, .2, .2, .2)$, and an increasing distribution $(.05, .1, .2, .3, .35)$, and varying assumptions on r to study the equilibria under different conditions. For concision, we'll only report a few of the more salient examples here. Generally, the simulations using constant per-date costs are consistent with the results in Section 4 assuming per date costs of $c\psi(q)$.

[28]We ignore mixed strategies.

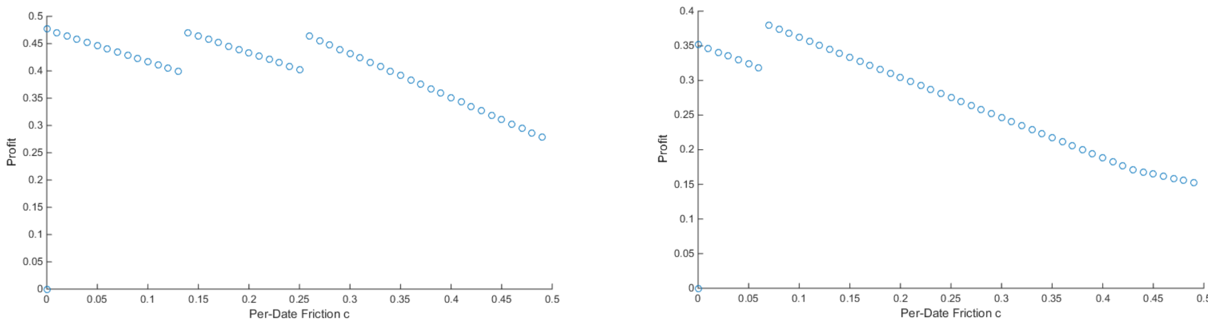

FIGURE 6.1. From left to right: a) increasing distribution, $r = .05$, $\alpha = 0$, b) increasing distribution, $r = .1$, $\alpha = 1$.

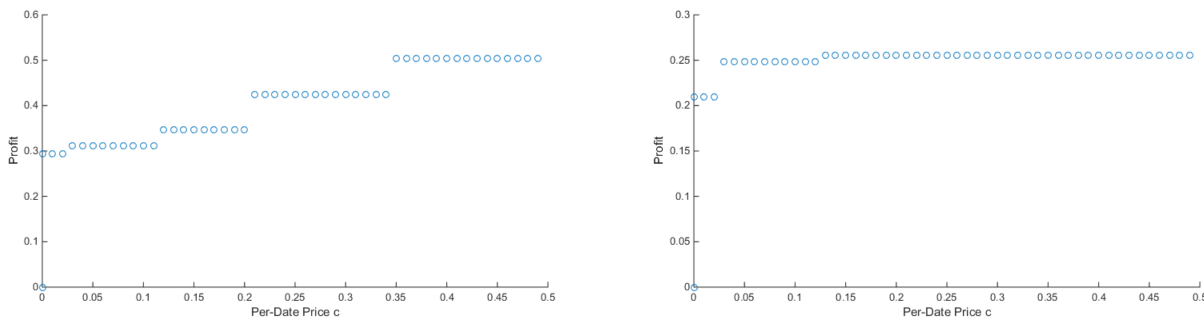

FIGURE 6.2. From left to right: a) increasing distribution, $r = .1$, $\alpha = 0$. b) uniform distribution, $r = .1$, $\alpha = 1$,

In Figure 6.1 a), we see the discrete type analogue to the frictional modular utility case for a monopolist discussed in Proposition 8. As per-date frictions increase, the price that can be charged to the lowest joining type decreases, and there is a negative direct effect on profit. However, higher type agents become less selective, and when the per-date friction is high enough to induce a higher type to accept the lowest joining type there is a discontinuous increase in profit due to the effective transfer from the high type to the cutoff type which counterbalances the direct effect. Thus we see multiple levels of c that are consistent with maximizing revenue, as in the previous analysis. b) shows the case with $\psi(q) = q$, but unlike the formal analysis of Proposition 8, per-date frictions are c instead of cq, meaning that the class structure will not hold in equilibrium and the aforemention proposition does not apply. However ,we see qualitatively similar results, with a negative direct effect of c on profit and discrete jumps back to higher profit when higher types accept the lowest joining type.

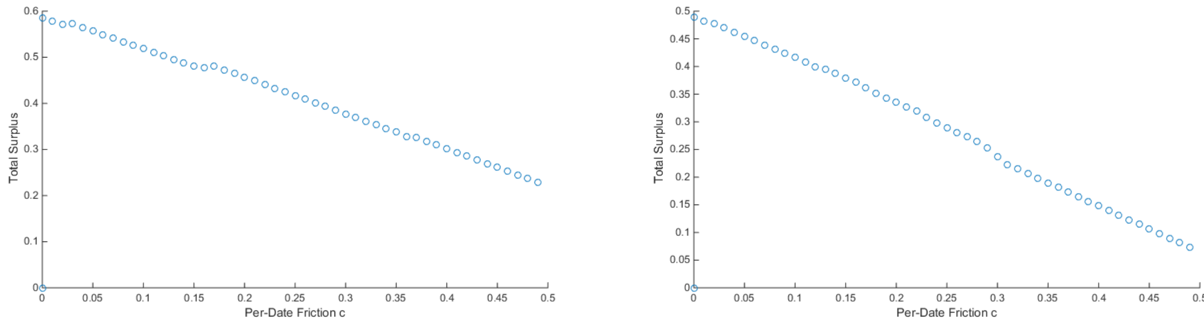

FIGURE 6.3. From left to right: a) increasing distribution, $r = .2$, $\alpha = 0$. b) increasing distribution, $r = .2$, $\alpha = 1$,

In Figure 6.2 a), we see the discrete type analogue to the per-date price modular utility case for a monopolist discussed in Corollary 3. This case is very similar to the frictional case, but raising per-date prices doesn't decrease the amount of surplus that can be extracted from the lowest joining type, so the effective transfers from high to low types as per-date prices increase are the only salient effect, and profit increases with per-date price. b) shows the case with supermodular utility $\psi(q) = q$, and per-date price c, a case which could not be studied before due the lack of a class structure. In fact, however, we see the same situation, where higher per-date prices increase profit, and even though the class basis for the claim of Corollary 3 does not hold, the argument that the IR-cutoff agent's utility can be extracted through a combination of fixed fees and per-date prices, and higher per-date prices should make higher types less selective and thus force them to match to the IR-cutoff type, increasing their expected match utility should still hold.

In Figure 6.3 a), we see the discrete type analogue to the frictional modular utility case for a social planner discussed in Section 4.1.2. As discussed before, the direct negative effect of increasing per-date frictions dominates, and higher frictions generally lower surplus, although small local increases are possible due to the non-endogenous lower bound of the rump class. In b), we see the case with $\psi(q) = q$, and per-date friction c, and the effect of increasing c is qualitatively similar.

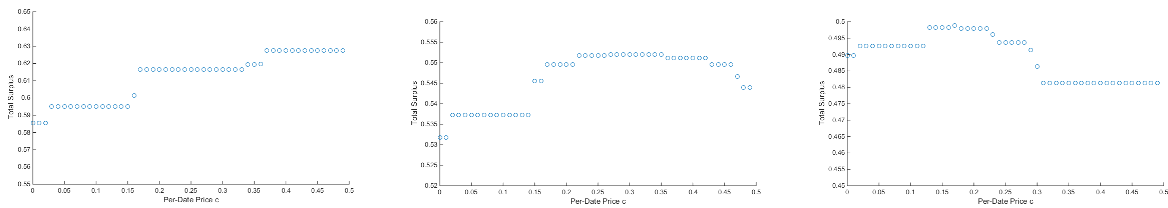

FIGURE 6.4. From left to right: a) increasing distribution, $r = .2$, $\alpha = 0$. b) increasing distribution, $r = .2$, $\alpha = .5$, c) increasing distribution, $r = .2$, $\alpha = 1$.

In Figure 6.4 a), we see the discrete type analogue to the per-date price modular utility case for a social planner discussed in Proposition 11. As discussed before, increasing per-date prices lowers the time costs of search, and because of modular utility sorting doesn't matter, so increasing per-date prices increases total surplus. We couldn't study optimal per-date prices with supermodular utility before due to tractability problems, but b) and c) we can examine numerical simulations of this case. As discussed before, with supermodular utility assortation increases surplus, so when per-date prices increase and agents become less picky, there will be a tradeoff between lowering time costs on the one hand and lowering sorting on the other. In fact, in b) with $\psi(q) = \sqrt{q}$ and moderate supermodularity we see exactly that, with total surplus initially increasing in per-date prices when decreasing time costs dominates and total surplus later decreasing when sorting effects dominate, yielding an optimal per-date price that is positive. With c), $\psi(q) = q$ and supermodularity is stronger. We see the same story here, but the optimal per-date price is significantly lower as the costs of lowering sorting are higher.

6.2. **Proofs.**

6.2.1. *Steady State–Linear Returns to Matching.* We'll now provide a proof of Proposition 5 via two lemmas. This closely follows Burdett and Coles, but requires some adjustment to accommodate per-date costs. First, we'll transcribe a useful result from Burdett and Coles. Define $\Gamma(x_1, x_2) \equiv (F(x_1) - F(x_2))^2 - f(x_2)\int_{x_2}^{x_1} F(x_1) - F(x)dx$

Lemma 10. *An increasing hazard rate f(x)/(1-F(x)) implies* $\Gamma(.) \geq 0$

Define $\phi(q(n), q(n-1)) \equiv \int_{q(n)}^{q(n-1)} \frac{F(q(n-1))-F(x)}{F(q(n-1))-F(q(n))} dx - c$ for $q(n-1) > \underline{q}$. Since F is strictly increasing and twice differentiable, ϕ is well defined, continuous and twice differentiable almost everywhere, restricting ourselves to right differentiation at the lower bound. It can be shown that $\phi \to -c$ as class size goes to zero and $\phi \to \underline{q} - q(n) - c$ as $q(n-1) \to \underline{q}$. Lemma 10 implies ϕ is decreasing in $q(n)$:

$\frac{\partial}{\partial q(n)} \int_{q(n)}^{q(n-1)} \frac{F(q(n-1))-F(x)}{F(q(n-1))-F(q(n))} dx - c = \int_{q(n)}^{q(n-1)} \frac{(F(q(n-1))-F(x))f(q(n))}{(F(q(n-1))-F(q(n)))^2} dx - 1 \leq 0.$

We can also show that ϕ is strictly increasing in $q(n-1)$:

$\frac{\partial}{\partial q(n-1)} \int_{q(n)}^{q(n-1)} \frac{F(q(n-1))-F(x)}{F(q(n-1))-F(q(n))} dx - c = \int_{q(n)}^{q(n-1)} \frac{(F(x)-F(q(n)))f(q(n))}{(F(q(n-1))-F(q(n)))^2} dx > 0.$

Fix N and let $q_n(N)$, $\lambda_n(N)$, $J(N)$ satisfy

i) $q_0(N) = \overline{q}$

ii) if $q_{n-1}(N) > \underline{q}$, $q_n(N) = \phi(q_n(N), q_{n-1}(N))\frac{N\lambda_n(N)}{r}$, $q_n(N) = \phi(q_n(N), q_{n-1}(N))\frac{N\lambda_n(N)}{r}$, $\lambda_n(N) = \sqrt{F(q_{n-1}(N)) - F(q_n(N))}/N$

iii) if $q_{n-1}(N) <= \underline{q}$, $q_n(N) = \lambda_n(N) = 0$

The following lemma shows inductively that each cutoff is well behaved if the previous one is. The main challenge is to show uniqueness, especially in the presence of a per-date cost. In (3.9), the LHS is (obviously) increasing, so if we can show the RHS is decreasing, uniqueness is guaranteed. Thus the meat of the proof is establishing the properties of the RHS.

Lemma 11. *If $q_{n-1}(N) > \underline{q}$ and is continuous at N for some N>0, then there is a unique solution for $q_n(N)$ if participation in search in class n can be supported, where $q_n(N)$ is continuous at N, $q_n(N) < q_{n-1}(N)$, $\lambda_n > 0$ and is continuous at N. q_n and λ_n go to zero as $q_{n-1} \to \underline{q}$. Additionally, $q_n(N)$ is increasing in $q_{n-1}(N)$,*

Proof. $\frac{\partial}{\partial q(n)} \frac{1}{r}(\phi(q(n), q(n-1))\sqrt{(F(q(n-1)) - F(q(n)))}) = \frac{1}{r}(\phi_1\sqrt{(F(q(n-1)) - F(q(n)))} - \phi\frac{f(q(n))}{\sqrt{(F(q(n-1))-F(q(n)))}})$. Consider the minimal c such that $\phi_1\sqrt{(F(q(n-1)) - F(q(n)))} - \phi\frac{f(q(n))}{\sqrt{(F(q(n-1))-F(q(n)))}} \geq 0$.

Then $c = \frac{1}{r}(\int_{q(n)}^{q(n-1)} \frac{F(q(n-1))-F(x)}{F(q(n-1))-F(q(n))} dx - \frac{(F(q(n-1))-F(q(n)))}{f(q(n))}(\int_{q(n)}^{q(n-1)} \frac{(F(q(n-1))-F(x))f(q(n))}{(F(q(n-1))-F(q(n)))^2} dx - 1))$. Then the expected payoff is $\frac{1}{r}(\frac{(F(q(n-1))-F(q(n)))}{f(q(n))}\sqrt{(F(q(n-1)) - F(q(n)))})\phi_1$. ϕ_1 must be negative and the remainder of the expression is positive, so the RHS of (3.9) must be negative. Thus, either $\frac{1}{r}(\phi_1\sqrt{(F(q(n-1))-F(q(n)))}-\phi\frac{f(q(n))}{\sqrt{(F(q(n-1))-F(q(n)))}})\leq 0$ and the RHS is decreasing while the LHS is increasing, ensuring a a unique solution, or c is high enough that any draw will be accepted ex-post, which also implies a unique cutoff. Direct inspection shows continuity given continuity of the constituent functions, and thus the continuity of $q_n(N)$ and $\lambda_n(N)$. The RHS is negative as $q_n \to q_{n-1}$ so $q_n < q_{n-1}$. Thus $\lambda_n(N) > 0$. $q_n \to 0$ goes to zero as $q_{n-1} \to \underline{q}$ since RHS is negative and $\lambda_n(N) \leq G(q_{n-1}) - G(\underline{q}) \to 0$ as $q_{n-1} \to \underline{q}$. Finally, $\frac{\partial}{\partial q(n-1)}\frac{1}{r}(\phi(q(n),q(n-1))\sqrt{(F(q(n-1))-F(q(n)))}) = \frac{1}{r}(q(n-1)-q(n)-c)\frac{f(q(n)}{2\sqrt{(F(q(n-1))-F(q(n)))}}$. Consider the minimal c such that $\frac{1}{r}(q(n-1)-q(n)-c)\frac{f(q(n)}{2\sqrt{(F(q(n-1))-F(q(n)))}} \leq 0$. Then $c = q(n-1)-q(n)$ and continuation value is $\frac{1}{G(q(n-1))-G(q(n))+r}\int_{q(n)}^{q(n-1)}(x-c)g(x)dx = \frac{1}{G(q(n-1))-G(q(n))+r}\int_{q(n)}^{q(n-1)}(x-q(n-1)+q(n))g(x)dx \leq \frac{1}{G(q(n-1))-G(q(n))+r}\int_{q(n)}^{q(n-1)}q(n)g(x)dx = \frac{G(q(n-1)_u)-G(q(n))}{G(q(n-1))-G(q(n))+r}q(n) < q(n)$. Thus $c \geq q(n-1)-q(n)$ ensures a corner solution for the cutoff, $\underline{q}$. Thus, q(n) is unchanging in q(n-1) if $c > q(n-1)-q(n)$, else the RHS is increasing in q(n-1), which, given that the LHS is unchanged and the LHS is increasing and the RHS decreasing in q(n), implies a rise in q(n-1) must induce a rise in q(n). □

We'll now prove Lemma 7. Note that (3.3) can be rewritten as $q(n,\hat{q}) = \frac{(N\lambda_n(\hat{q})E[q'|n,\hat{q},match]-N\gamma_n(\hat{q})c)}{N\lambda_n(\hat{q})+r} = \frac{N\lambda_n(\hat{q})(E[q'|n,\hat{q},match]-c)}{N\lambda_n(\hat{q})+r} - \frac{N(\gamma_n(\hat{q})-\lambda_n(\hat{q}))c)}{N\lambda_n(\hat{q})+r}$ This is simply (3.4) minus c times a scalar on the RHS and with the values of $\gamma_n = Pr[date|n,\hat{q}]$, $\lambda_n = Pr[match|n,\hat{q}]$ and $E[q'|n,\hat{q},match]$ changed to reflect that agents may reject dates inside their class and accept agents outside their class, changing their probability of dating and matching over a given interval and changing the distribution of matches accepted. Every agent in a class must have the same expected match quality, so it will suffice to consider $q(n) = \frac{N\lambda_n(E[q'|n,match]-c)}{N\lambda_n+r} - \frac{N(\gamma_n-\lambda_n)c)}{N\lambda_n+r}$.

Lemma 12. *Given LRM and 1-F log-concave, a class n starting at q(n-1) where the probability of an agent in n rejecting a date inside the class is strictly positive must have* $\frac{N\lambda_n(E[q'|n,match]-c)}{N\lambda_n+r} \leq q_E(n_{LPPE,q(n)})$.

Proof. Forthcoming. □

We can now prove Lemma 7:

Proof. Let k be the first class with a positive measure of agents deviating from the LPPE. Lemma 12 shows that $\frac{N\lambda_k(E[q'|k,match]-c)}{N\lambda_k+r} \leq q_E(k_{LPPE,q(k)})$, with strict inequality holding if the probability of rejecting a match within one's class is strictly positive, and $\frac{N(\gamma_k-\lambda_k)c)}{N\lambda_k+r} \geq 0$, with strict inequality holding if the probability of accepting a match outside one's class is strictly positive. Then, since $q_E(q(k)) = q(k)$, $q_E(k) < q_E(k_{LPPE,q(k)}) \leq q_E(LPPE, q(k-1)) \leq q_E(q(k))$. Contradiction. We can proceed inductively from here. Suppose that $q(n-1) \leq q(LPPE, n-1)$. Then $q(n) \leq q(LPPE, q(n-1))$ as before. Lemma 11 establishes that $q(LPPE, q(n-1)) \leq q(LPPE, n)$, so $q(n-1) \leq q(LPPE, n)$. □

6.2.2. *Steady State–Constant Returns to Matching.* In addition to LRM, we can also study the analogous model with constant returns to matching. The CRM analysis largely follows Burdett and Coles (1997). While several proofs must be amended to account for per-date costs, some go through unchanged. Define the distribution of agents leaving the platform by $H(q)$ and the mass of agents leaving the platform by O.

Within a given class, we can get a simple characterization of outflow. Outflow in a class is given by the number of agents on the platform, N, times the proportion of agents in the class, λ_n, times the probability of an agent in the class drawing another agent in that class, λ_n. Then outflow from class n is $\lambda_n^2 N$. Then, in an LSSE,

$$\lambda_n = \sqrt{(F(q(n-1)) - F(q(n)))/N} \tag{6.3}$$

We also have that, for any $[z_1, z_2)$ in class n, $\lambda_n(G(z_2) - G(z_1))N = F(z_2) - F(z_1)$ and thus, with the differentiability of F,

$$g(q) = \frac{f(q)}{\lambda_n N} \tag{6.4}$$

Thus the density of agents on the platform in a given class is inflow density times a scalar. Combining equation 3.5 and balanced flow, we can get eliminate G terms, yielding class cutoffs solely in terms of inflows, N, and c.

$$q(n) = \frac{1}{r}\left(\int_{q(n)}^{q(n-1)} \frac{F(q(n-1)) - F(x)}{F(q(n-1)) - F(q(n))} dx - c\right)\sqrt{(F(q(n-1)) - F(q(n)))/N} \tag{6.5}$$

We're can now characterize the LSSE in this environment:

Proposition 12. *Given F, (G, N) defines a LSSE if and only if G satisfies (6.4) and $\{(\lambda_n, q(n))\}_{n=0}^{J}$ satisfies (6.3), (6.5), $q(0) = \overline{q}$, $q(J) <= \underline{q}$, and $\sum_n \lambda_n = 1$.*

Proof. $\sum_n \lambda_n = 1$, the boundary conditions, and (6.3)-(6.5) are necessary in an LSSE by construction. Conversely, the assumptions guarantee $G(\overline{q}) = 1$, $G(\underline{q}) = 0$ and G increasing, so G is a well defined steady state distribution and any G and N satisfying them form a valid LSSE. □

To ensure existence of an LSSE, we'll need to make some distributional assumptions. The increasing hazard rate will ensure that, for each possible N, the class structure is unique. We'll now provide a proof of Proposition 13 via a lemma. This closely follows Burdett and Coles, but requires some adjustment to accommodate per-date costs.

Lemma 10 implies ϕ is decreasing in $q(n)$:

$\frac{\partial}{\partial q(n)} \int_{q(n)}^{q(n-1)} \frac{F(q(n-1))-F(x)}{F(q(n-1))-F(q(n))} dx - c = \int_{q(n)}^{q(n-1)} \frac{(F(q(n-1))-F(x))f(q(n))}{(F(q(n-1))-F(q(n)))^2} dx - 1 \leq 0.$

We can also show thatϕ is strictly increasing in q(n-1):

$\frac{\partial}{\partial q(n-1)} \int_{q(n)}^{q(n-1)} \frac{F(q(n-1))-F(x)}{F(q(n-1))-F(q(n))} dx - c = \int_{q(n)}^{q(n-1)} \frac{(F(x)-F(q(n)))f(q(n))}{(F(q(n-1))-F(q(n)))^2} dx > 0.$

Fix N and let $q_n(N)$, $\lambda_n(N)$, $J(N)$ satisfy

i) $q_0(N) = \overline{q}$

ii) if $q_{n-1}(N) > \underline{q}$, $q_n(N) = \phi(q_n(N), q_{n-1}(N))\frac{\delta\lambda(N)}{r}$, $q_n(N) = \phi(q_n(N), q_{n-1}(N))\frac{\lambda_n(N)}{r}$, $\lambda_n(N) = \sqrt{F(q_{n-1}(N)) - F(q_n(N))/N}$

iii) if $q_{n-1}(N) <= \underline{q}$, $q_n(N) = \lambda_n(N) = 0$

The following Lemma shows inductively that each cutoff is well behaved if the previous one is. The main challenge is to show uniqueness, especially in the presence of a per-date cost. In 2.8, the LHS is (obviously) increasing, so if we can show the RHS is decreasing, uniqueness is guaranteed. Thus the meat of the proof is establishing the properties of the RHS.

Lemma 13. *If $q_{n-1}(N) > \underline{q}$ and is continuous at N for some N>0, then there is a unique solution for $q_n(N)$, where $q_n(N)$ is continuous at N, $q_n(N) < q_{n-1}(N)$, and $\lambda_n > 0$ and is continuous at N. $q_n(N)$ and $\lambda_n(N)$ go to zero as $q_{n-1}(N) \to \underline{q}$. Additionally, $q_n(N)$ is increasing in $q_{n-1}(N)$,*

Proof. $\frac{\partial}{\partial q(n)}\frac{1}{r}(\phi(q(n),q(n-1))\sqrt{(F(q(n-1))-F(q(n)))/N}) = \frac{1}{r}(\phi_1\sqrt{(F(q(n-1))-F(q(n)))/N} - \phi\frac{f(q(n))}{\sqrt{N(F(q(n-1))-F(q(n)))}})$. Consider the minimal c such that $\phi_1\sqrt{(F(q(n-1))-F(q(n)))/N} - \phi\frac{f(q(n))}{\sqrt{N(F(q(n-1))-F(q(n)))}} \geq 0$.

Then $c = \int_{q(n)}^{q(n-1)} \frac{F(q(n-1))-F(x)}{F(q(n-1))-F(q(n))}dx - \frac{(F(q(n-1))-F(q(n)))}{f(q(n))}(\int_{q(n)}^{q(n-1)} \frac{(F(q(n-1))-F(x))f(q(n))}{(F(q(n-1))-F(q(n)))^2}dx - 1)$. Then the RHS of (6.5) is $\frac{1}{r}(\frac{(F(q(n-1))-F(q(n)))}{f(q(n))}\sqrt{(F(q(n-1))-F(q(n))/N})\phi_1$. By Lemma 10, ϕ_1 must be negative and the remainder of the expression is positive, so expected payoff must be negative. Thus, either $\frac{1}{r}(\phi_1\sqrt{(F(q(n-1))-F(q(n)))/N} - \phi\frac{f(q(n))}{\sqrt{N(F(q(n-1))-F(q(n)))}}) \leq 0$ and the RHS is decreasing while the LHS is increasing, ensuring a unique solution, or c is high enough that any draw will be accepted ex-post, which also implies a unique cutoff. Direct inspection shows continuity given continuity of the constituent functions, and thus the continuity of $q_n(N)$ and $\lambda_n(N)$. The RHS is negative as $q_n \to q_{n-1}$ so $q_n < q_{n-1}$. Thus $\lambda_n(N) > 0$. $q_n \to 0$ goes to zero as $q_{n-1} \to \underline{q}$ since RHS is negative and $\lambda_n(N) \leq G(q_{n-1}) - G(\underline{q}) \to 0$ as $q_{n-1} \to \underline{q}$. Finally, $\frac{\partial}{\partial q(n-1)}\frac{1}{r}(\phi(q(n),q(n-1))\sqrt{(F(q(n-1))-F(q(n)))/N})$= $\frac{1}{r}(q(n-1)-q(n)-c)\frac{f(q(n)}{2\sqrt{N(F(q(n-1))-F(q(n)))}}$. Consider the minimal c such that $\frac{1}{r}(q(n-1)-q(n)-c)\frac{f(q(n)}{2\sqrt{N(F(q(n-1))-F(q(n)))}} \leq 0$. Then $c = q(n-1)-q(n)$ and continuation value is $\frac{1}{G(q(n-1))-G(q(n))+r}\int_{q(n)}^{q(n-1)}(x-c)g(x)dx = \frac{1}{G(q(n-1))-G(q(n))+r}\int_{q(n)}^{q(n-1)}(x-q(n-1)+q(n))g(x)dx \leq$ $\frac{1}{G(q(n-1))-G(q(n))+r}\int_{q(n)}^{q(n-1)}q(n)g(x)dx$= $\frac{G(q(n-1)_u)-G(q(n))}{G(q(n-1))-G(q(n))+r}q(n) < q(n)$. Thus $c \geq q(n-1) - q(n)$ ensures a corner solution for the cutoff, $\underline{q}$. Thus, q(n) is unchanging in q(n-1) if $c >$

$q(n-1)-q(n)$, else the RHS is increasing in q(n-1), which, given that the LHS is unchanged and the LHS is increasing and the RHS decreasing in q(n), implies a rise in q(n-1) must induce a rise in q(n). □

Proposition 13. *For all N>0, there exist unique, continuous solutions for $q_n(N)$ and $\lambda_n(N)$ satisfying (6.3)-(6.5), $q_0(N)=\overline{q}$ and $q_J(N)\leq \underline{q}$. such that $q_n(N)<q_{n-1}(N)$ and $\lambda_n(N)>0$ if $q_{n-1}(N)>\underline{q}$.*

Proof. For the base case of $q_0=\overline{q}$, q_0 is a constant function of N. Lemma 13 ensures that $q_{n-1}(N)$ continuous implies $q_n(N)$ continuous, and $q_n(N)<q_{n-1}(N)$, so induction follows. Continuity of $\lambda_n(N)$ follows from the continuity of $q_n(N)$, $q_{n-1}(N)$, and $\sqrt{(F(x)-F(y)/N}$. $\lambda_n(N)>0$ follows from the fact that $q_n(N)<q_{n-1}(N)$ and $f(q)>0$ for $q\in[\underline{q},\overline{q}]$. □

However, N may not be consistent with $G(\overline{q})=1$, so we'll need an additional result. Log-concavity ensures the continuity of class sizes, and that, along with values of N inducing values $G(\overline{q},N)$ above and below 1, ensures the existence of $\{\lambda_n\}_{n=1}^{J}$ such that $\sum_n \lambda_n=1$. It's worth noting that the inclusion of a per-date cost makes uniqueness of the cutoffs harder to obtain than in the Burdett and Coles environment–without per date costs, the agent's optimization problem has convenient monotonicity properties that per-date costs militate against. However, the already necessary assumption of log-concavity of the survivor function also eliminates cases where per-date costs could induce a multiplicity of cutoffs.

Proposition 14. *An LSSE exists.*

Proof. Proposition 13 guarantees this result so long as $\sum_n \lambda_n(N)=1$. $\sum_n \lambda_n(N)=\frac{1}{\sqrt{N}}\sum_n\sqrt{F(q_{n-1}(N))-F(q_n(N))}>\frac{1}{\sqrt{N}}\sum_n F(q_{n-1}(N))-F(q_n(N))=\frac{1}{\sqrt{N}}(F(\overline{q})-F(\underline{q}))=\frac{1}{\sqrt{N}}$, so $\lim\limits_{N\to 0}\sum_n\lambda_n(N)=\infty$. $\sum_n\lambda_n(N)$

$\Rightarrow q(1)=\frac{1}{r}(\int_{q_1(N)}^{\overline{q}}\frac{1-F(x)}{1-F(q_1(N))}dx-c)\sqrt{1-F(q_1(N)))/N}$

$\frac{1-F(x)}{1-F(q_1(N))}<1$ if $x\in(q_1(N),\overline{q}]$ so

$\int_{q_1(N)}^{\overline{q}}\frac{1-F(x)}{1-F(q_1(N))}dx<\overline{q}-q_1(N)$.

Then we have

$q_1(N) < \frac{1}{r}(\overline{q} - q_1(N) - c)\sqrt{(1 - F(q_1(N))/N}$

$q_1(N)\frac{1+r}{r} < \frac{1}{r}(\overline{q} - c)\sqrt{(1 - F(q_1(N)))/N}$

$q_1(N) < \frac{1}{1+r}(\overline{q} - c)\sqrt{(1 - F(q_1(N)))/N}$

so $q_1(N) \to 0$. For N sufficiently large, $q_1(N) < \underline{q}$, so $F(q_1(N)) = 0$. Then

$\sum_n \sqrt{F(q_{n-1}(N)) - F(q_n(N))} = \sqrt{F(q_0(N)) - F(q_1(N))} = 1$

$\sum_n \lambda_n(N) = \frac{1}{\sqrt{N}} \to 0.$

$\lambda_n(N)$ is continuous for all n, so $\sum_n \lambda_n(N)$ is continuous. Then, given $\sum_n \lambda_n(N) > 1$ for some N and $\sum_n \lambda_n(N) < 1$ for some N, the IVT ensures an N exists such that $\sum_n \lambda_n(N) = 1$. □

Finally, we'd like to have uniqueness. This will require further distributional assumptions. Burdett and Coles only need that $xf(x)$ is increasing, but the inclusion of per-date costs again imposes stronger requirements for uniqueness. Unfortunately, in this case their assumptions are not strong enough to resolve the monotonicity issues with per-date costs. For sufficiently small per-date costs, the increasing $xf(x)$ assumption is adequate, but to ensure uniqueness for any per-date cost we'll need the stronger assumption that $f(x)$ is increasing. This assumption is quite onerous, so we'll stick with the weaker assumption from Burdett and Coles and focus on sufficiently small per-date costs in the later analysis.

Lemma 14. *$q_n(N)$ is decreasing and differentiable in N.*

Proof. Differentiability follows from induction on (6.5). For the first class, we must have $q_1(N) = \frac{1}{r}(\int_{q_1(N)}^{\overline{q}} \frac{1-F(x)}{1-F(q_1(N))}dx - c)\sqrt{1 - F(q_1(N)))/N}$. Denote the LHS L and the RHS R. Lemma 13 shows $R_{q_1} = \frac{\partial}{\partial q_1(N)}\frac{1}{r}(\int_{q_1(N)}^{\overline{q}} \frac{1-F(x)}{1-F(q_1(N))}dx - c)\sqrt{1 - F(q_1(N)))/N}$ is negative while $L_1 = 1$ is positive. Suppose $q_1(N)$ is weakly increasing in N for some N. Then $L_N = q_1^{'}(N) > 0$ and $R_N = q_1^{'}(N)R_{q_1} - R/(2N)$ is negative. But L=R. Contradiction. We can now procede inductively. Suppose $q_{n-1}(N)$ is decreasing in N. (6.5) must hold, and $R_{q_n} = \frac{\partial}{\partial q_n(N)}\frac{1}{r}(\int_{q_n(N)}^{q_{n-1}(N)} \frac{F(q_{n-1}(N))-F(x)}{F(q_{n-1}(N))-F(q_n(N))}dx - c)\sqrt{(F(q_{n-1}(N)) - F(q_n(N)))/N}$ is negative and $R_{q_{n-1}} = \frac{\partial}{\partial q_{n-1}(N)}\frac{1}{r}(\int_{q_n(N)}^{q_{n-1}(N)} \frac{F(q_{n-1}(N))-F(x)}{F(q_{n-1}(N))-F(q_n(N))}dx - c)\sqrt{(F(q_{n-1}(N)) - F(q_n(N)))/N}$ is positive by Lemma 13. Then the $L_N = q_n^{'}(N) > 0$ and $R_N = q_n^{'}(N)R_{q_n} + q_{n-1}^{'}(N)R_{q_{n-1}} - R/(2N)$, which is negative since $q_{n-1}^{'}(N)$ is negative. □

Lemma 15. *$\lambda_{n-1} \geq \lambda_n$ for any N>0 with $xf(x)$ for c sufficiently small or any c with $f(x)$ increasing.*

Proof. Differentiability follows from induction on (6.5). For the first class, we must have $q_1(N) = \frac{1}{r}(\int_{q_1(N)}^{\overline{q}} \frac{1-F(x)}{1-F(q_1(N))}dx - c)\sqrt{1-F(q_1(N)))/N}$. Denote the LHS L and the RHS R. Lemma 13 shows $R_{q_1} = \frac{\partial}{\partial q_1(N)}\frac{1}{r}(\int_{q_1(N)}^{\overline{q}} \frac{1-F(x)}{1-F(q_1(N))}dx - c)\sqrt{1-F(q_1(N)))/N}$ is negative while $L_1 = 1$ is positive. Suppose $q_1(N)$ is weakly increasing in N for some N. Then $L_N = q_1^{'}(N) > 0$ and $R_N = q_1^{'}(N)R_{q_1} - R/(2N)$ is negative. But L=R. Contradiction. We can now procede inductively. Suppose $q_{n-1}(N)$ is decreasing in N. (6.5) must hold, and $R_{q_n} = \frac{\partial}{\partial q_n(N)}\frac{1}{r}(\int_{q_n(N)}^{q_{n-1}(N)} \frac{F(q_{n-1}(N))-F(x)}{F(q_{n-1}(N))-F(q_n(N))}dx - c)\sqrt{(F(q_{n-1}(N)) - F(q_n(N))))/N}$ is negative and $R_{q_{n-1}} = \frac{\partial}{\partial q_{n-1}(N)}\frac{1}{r}(\int_{q_n(N)}^{q_{n-1}(N)} \frac{F(q_{n-1}(N))-F(x)}{F(q_{n-1}(N))-F(q_n(N))}dx - c)\sqrt{(F(q_{n-1}(N)) - F(q_n(N))))/N}$ is positive by Lemma 13. Then the $L_N = q_n^{'}(N) > 0$ and $R_N = q_n^{'}(N)R_{q_n} + q_{n-1}^{'}(N)R_{q_{n-1}} - R/(2N)$, which is negative since $q_{n-1}^{'}(N)$ is negative. □

Lemma 16. *$xf(x)$ strictly increasing in x guarantees $\lambda_{n-1} \geq \lambda_n$ for c such that $c' \geq c > 0$ for some c'. $f(x)$ increasing guarantees $\lambda_{n-1} \geq \lambda_n$ for any c.*

Proof. Trivial for $n-1 \geq J(N)$. For $n-1 < J(N)$, we first want to show $\lambda_{n-1} \geq \lambda_n$ for all n. Define $\theta(q_l, q_h) = \int_{q_l}^{q_h} \frac{F(q_h)-F(x)}{F(q_h)-F(q_l)}dx - c)\sqrt{(F(q_h) - F(q_l))}$. Thus $\frac{1}{\sqrt{N}r} = q_l/\theta(q_l, q_h)$. $\lambda = \sqrt{(F(q_h) - F(q_l))/N}$ so it will suffice to show $\frac{\partial}{\partial q_h}F(q_h) - F(q_l(q_h))$ is increasing. The implicit function theorem yields $\frac{\partial}{\partial q_h}F(q_h) - F(q_l(q_h)) = f(q_h) - f(q_l)\frac{\frac{\delta}{\sqrt{N}(1-\delta)}\theta_2}{1-\frac{\delta}{\sqrt{N}(1-\delta)}\theta_1} = f(q_h) - f(q_l)\frac{q_l\theta_2/\theta}{1-q_l\theta_1/\theta}$. We can show that this is non-negative if and only if $\int_{q_l}^{q_h} x\,f(x) - q_l\,f(q_l) - c\,dx \geq 0$. If c is sufficiently small, this will be satisfied (clearly always satisfied for c=0 and $xf(x)$ increasing.) If $xf(x)$ strictly increasing, a strictly positive c can be supported. It can be shown that if $\frac{(q_h-q_l)f(q_l)}{F(q_h)-F(q_l)} \leq 1$, any c large enough to violate the above inequality will yield a corner solution for any agent's optimization problem, with agents accepting any match, a single class, and uniqueness thus ensured. if $f(x)$ is increasing, $\frac{(q_h-q_l)f(q_l)}{F(q_h)-F(q_l)} \leq 1$. □

Proposition 15. *$xf(x)$ strictly increasing in x guarantees the existence of a unique LSSE for all c such that $c' \geq c > 0$ for some c'. $f(x)$ increasing guarantees uniqueness for any c.*

Proof. Total differentiation of $\lambda_{n-1}(N) = \sqrt{F(q_{n-1}(N)) - F(q_n(N))/N}$ yields $\frac{q'_{n-1}(N)f(q_{n-1}(N)) - q'_n(N)f(q_n(N))}{2\lambda_n N} - \frac{\lambda_n}{2N}$ for all but the last class and $\frac{q'_{n-1}(N)f(q_{n-1}(N))}{2\lambda_n N} - \frac{\lambda_n}{2N}$ for the last class. Summing over n, we have $-\frac{\lambda_J}{2N} + \sum_{n=1}^{J(N)-1} (\frac{1}{2N}(\frac{1}{\lambda_{n+1}} - \frac{1}{\lambda_n})q'_n(N)f(q_n(N)) - \frac{\lambda_n}{2N})$. λ_n are decreasing in n by Lemma 16, so $\frac{1}{\lambda_{n+1}} - \frac{1}{\lambda_n}$ increasing, and $q'_n(N)$ decreasing by Lemma 15. Thus the sum is negative, proving that $\sum_n \lambda_n$ is strictly decreasing in N. Thus, the N such that $\sum_n \lambda_n = 1$ must be unique, and so the LSSE. □

6.2.3. *Strategic Platforms.* We'll now prove Proposition 9. Define $\lambda_F(n) \equiv F(q(n-1)) - F(q(n))$, the inflow mass in class n. Suppose $\psi(q) = q^a$ and define the mean ψ value in class n as $m_\psi(n) \equiv \int_{q(n)}^{q(n-1)} q^a \frac{f(q)}{\lambda_F(n)} dq$. Define the length of class n as $l(n) \equiv q(n-1) - q(n)$.

Proof. $\frac{\partial TS}{\partial c} = \sum_{n=1}^{J} q_{Ec}(n,c) m_\psi(n) \lambda_F(n) +$
$q_E(n,c)\left(q_c(n-1,c)q(n-1,c)^\alpha f(q(n-1,c)) - q_c(n,c)q(n,c)^\alpha f(q(n,c))\right)$

Using the fact that $q_E(n,c) = q(n,c)$ for all but the last class J, manipulating the summation, and suppressing c, we have

$\frac{\partial TS}{\partial c} = \sum_{n=1}^{J-2} q_c(n)\left(m_\psi(n) \cdot \lambda_F(n) - q(n)^\alpha \cdot f(q(n)) \cdot l(n+1)\right)$
$+ q_c(J-1)\left(m_\psi(J-1) \cdot \lambda_F(J-1) - q(J-1)^\alpha \cdot f(q(J-1))(q(J-1) - q_E(J))\right)$
$+ q_c(J)\left(m_\psi(J) \cdot \lambda_F(J) - q(J)^\alpha \cdot f(q(J)) \cdot q_E(J)\right)$

If every term in this summation is negative, $\frac{\partial TS}{\partial c}$ is negative. By Proposition 7, $q_c(n)$ is decreasing, so it suffices to show $m_\psi(n) \cdot \lambda_F(n) > q(n)^\alpha \cdot f(q(n)) \cdot l(n+1)$ for each n<J-1, and the corresponding inequalities for J-1 and J. By Jensen's inequality, $m_\psi(n) > E[q|q \in [q_n, q_{n-1})]^\alpha$. Given that F has full support, $E[q|q \in [q_n, q_{n-1})]^\alpha > q_n$. Thus, as $\alpha \to \infty$, $E[q|q \in [q_n, q_{n-1})]^\alpha / q^\alpha \to \infty$. Then $m_\psi(n) \cdot \lambda_F(n) > q(n)^\alpha \cdot f(q(n)) \cdot l(n+1)$ for α sufficiently high, and since J is finite, an α exists ensuring $m_\psi(n) \cdot \lambda_F(n) > q(n)^\alpha \cdot f(q(n)) \cdot l(n+1)$ for all $n < J-1$, as well as the inequalities for J-1 and J. □

We'll now study a selection of simulations. We focus on the modular utility case, and find that increasing c typically decreases total surplus. This is not surprising given that a

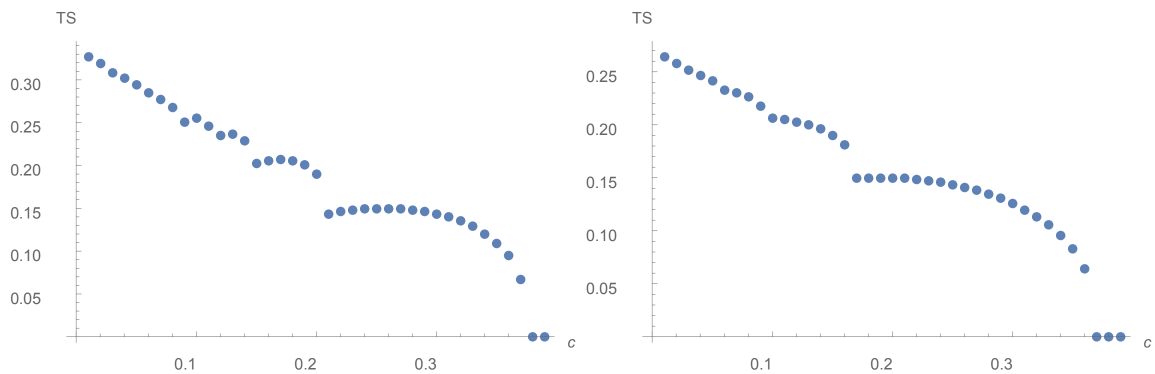

FIGURE 6.5. From left to right: a) $F(x) = \sqrt{x - \underline{q}}/\sqrt{1 - \underline{q}}$, $r = .001$, $\underline{q} = .01$ b) $F(x) = \sqrt{x - \underline{q}}/\sqrt{1 - \underline{q}}$, $r = .1$, $\underline{q} = .01$

per-date friction of c decreases each agent's payoff by c. However, efficiency gains due to less selective agents and changes in the rump class due to c can outweigh the direct cost of c in some cases. In Figure 6.5, we assume a decreasing density f. This puts more weight on the rump class, creating more periodicity in the surplus as the class cutoffs shift downward in c and the

rump class goes from being a large, relatively efficient class to a small class whose size is limited by the support of the distribution, and then to a large class again as the last class cutoff passes the bottom of the support of the distribution and the next class becomes the last class. In both the case where r=.1 and r=.001, we clearly see the periodic component to total surplus, and over some intervals total surplus is actually increasing in c. Generally, the degree of discounting doesn't make a large difference unless agents are very impatient.

In Figure 6.7 a), we again assume a decreasing density f, but assume the a smaller support for f. This largely eliminates the periodic component to total surplus and leaves only the direct effect of c–total surplus is approximately linearly decreasing in c. Because the lowest type agents in the distribution are still half the quality of the highest types, there are far fewer endogenous classes, and efficiency losses due to excessive selectivity are lower. Thus, efficiency gains due to increasing c are much less relevant. In Figure 6.7 b) we consider an analogous case with a uniform distribution. The factors that could lead to increasing total

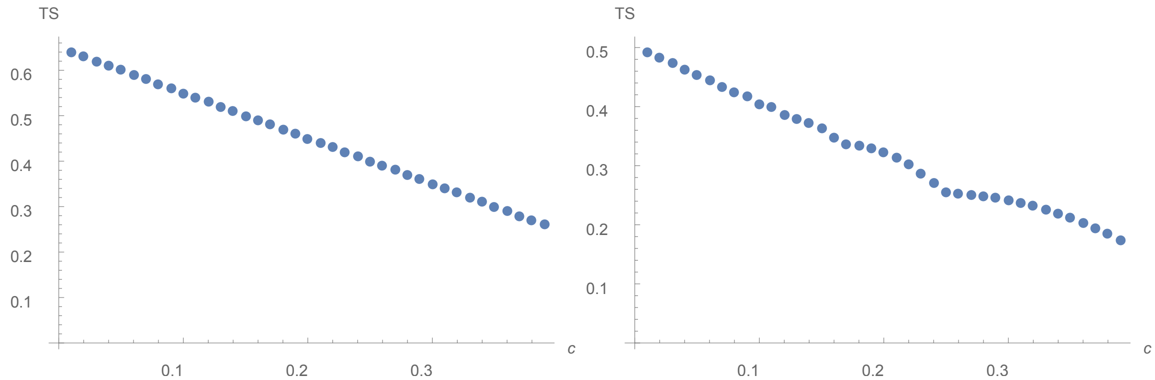

FIGURE 6.6. From left to right: a) $F(x) = \sqrt{x - \underline{q}}/\sqrt{1 - \underline{q}}$, $r = .001$, $\underline{q} = .5$ b) $F(x) = (x - \underline{q})/(1 - \underline{q})$, $r = .001$, $\underline{q} = .01$

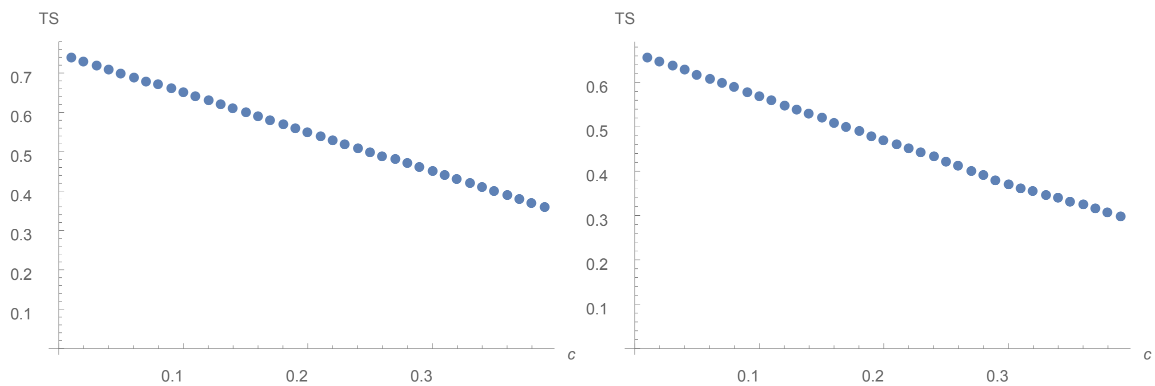

FIGURE 6.7. From left to right: a) $F(x) = (x - \underline{q})/(1 - \underline{q})$, $r = .001$, $\underline{q} = .5$, $\underline{q} = .001$ b) $F(x) = (x - \underline{q})^2/(1 - \underline{q})^2$, $r = .001$, $\underline{q} = .01$

surplus are weaker in this case, but we still see some periodic effect and a small region where total surplus is slightly increasing in c.

In Figure 6.7 a), we see the uniform distribution case corresponding to Figure 6.7 a). Again, the narrow range of qualities yields a monotonically decreasing total surplus. Finally, in Figure 6.7 b) we see total surplus when f is increasing. Here, even with a lower limit of support close to zero the second order effects are dominated by the linear friction costs.

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