Quality Versus Fit: Market Design and Externalities on Multidimensional Matching Platforms

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Abstract

This paper studies externalities in one-to-one matching markets when agents have preferences over multidimensional types by utilizing a minimal search setting where agents are either high or low quality and have an idiosyncratic per-match "fit" shock. It identifies a novel source of externalities that does not exist in the one-dimensional models focused on in the previous literature, but is endemic in multidimensional settings, appearing in both search and frictionless matching models so long as nontransferabilities are present. Agents match too aggressively on traits where preferences are homogeneous across agents (*quality*), and too little on traits where preferences are heterogeneous across agents (*fit*). This effect is decomposed into an *intermatch externality* – when you match to someone, you impose a cost on the rest of the market by removing them from it, and an *intramatch externality* – you don't account for your partner's payoffs when choosing partners. Given these generic externalities, we provide a survey of instruments a matching platform could use to improve surplus, analyzing for each the efficiency properties of the solution and its ease of implementation under a variety of assumptions. These instruments include having the platform act as a middle-man to make the transfers that agents cannot make directly make, utilizing two-part tariffs, splitting the platform along quality, and censoring agents' choice sets (i.e. curating the set of partners they can see on the platform).

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1 Introduction

Consider a simple matching model with two newly minted doctors and two residency programs, each with one opening. Both the doctors and the programs have preferences over *quality*. Program A is ranked first while Program B is ranked second, and student a is ranked first while student b is ranked second. However, there is also another form of variation, *fit*. Student a and program B are specialized in gynaecology, while student b and program A are specialized in nephrology. An obvious question for society, a medical association, or a for-profit matching platform that serves the medical labor market is the following: which student should go to which program? More formally, if each potential pairing generates a match surplus (lives saved, disability adjusted life years, etc.), how do we maximize the total surplus in the market? For simplicity, let's say that each agent's quality contributes an additively separable payoff to the match surplus. Then, to maximize total surplus, we should assign a to B and b to A since the total quality in the market is same regardless, but a has a much better fit with B, and b has a much better fit with A. However, if salaries for each program are non-negotiable, then for appropriate relative weights on quality preference – that is, if both programs and students have a strong preference for prestige – the stable assignment will see a matching to A and b matching to B. The lost surplus relative to assignment $\{aB,bA\}$ arises from wedges between the preferences of the individuals in the market and those of the planner–matching externalities.

In this paper, we study these efficiency issues in two-sided one-to-one matching models – markets with two sides, each having a preference over the other, where each agent searches for a single partner. These models can be used to study marriage and dating, job search, and the assignment of students to schools, among other things. There is a rich literature on externalities in these markets, but it focuses on stylized one-dimensional preference models, typically assuming vertical preferences, where every agent agrees on the ranking of all agents on the other side of the market. This paper extends the literature by developing a very general model of matching over multiple partner traits that is simple enough to illustrate a novel source of externalities that, while absent in one-dimensional models, is endemic to matching models with multi-dimensional preferences. In particular, we show that tradeoffs between traits embodying quality and traits embodying fit generate externalities in a wide variety of settings. By quality, we mean that preferences over the trait are homogeneous, with all agents agreeing on the preference ordering of potential partners. Examples of quality variables in job search include prestige, ranking, and salary. Examples in dating and marriage include income, attractiveness, and status. By contrast, a trait embodying fit exhibits heterogeneous preferences – different agents have different, idiosyncratic preference orderings. In job search, location and specialization can be framed as fit variables. Examples in dating and marriage include location, tastes, and personal chemistry. Notably, horizontal preferences á la Hotelling (1929) and Salop (1979) are a common form of fit in a variety of applications. Having established these externalities, we study a variety of possible instruments to improve efficiency.

We focus on a search model of matching, where agents meet potential partners (who they can accept or reject) via a Poisson process. We allow the meeting rate to be either constant, termed *constant returns to matching* (CRM), or to be proportional to the number of agents on the platform, termed *linear returns to matching* (LRM). CRM illustrates a search market where the bottleneck on search is the agent's ability to evaluate parters – no matter how many people an agent meets, they can only evaluate a fixed number (say, 10) in a day. This corresponds to a platform like Tinder in its early years, where users recieved a stream of largely uncurated draws (besides filtering for location). LRM illustrates a search market where the bottleneck on search is the availability of partners, implying that a larger platform then means more opportunities to match. This corresponds to traditional online dating websites like Match.com where users can search and filter results without restriction, as well as apps like Coffee Meets Bagel that curate draws. We allow for both of the common search friction specifications: time discounting and search costs that impose a cost for each meeting. For simplicity, we model quality via a binary vertical trait θ that determines the quality payoff recieved by their partner. Agents are either Studs (i.e. the high type, abbreviated H) or Duds (i.e. the low type, abbreviated L), where $\theta_H > \theta_L$. Fit is modeled as an idiosyncratic match shock $\psi \sim F(.)$ where every possible pairing has an associated mutual fit parameter. Agents permanently leave the market when they match.

When two individuals both accept a match, they generate a match surplus. If the agents can costlessly bargain over this surplus, we term it a *transferable utility (TU)* matching market. As with the Coase Theorem in one-sided markets, TU tends to resolve matching externalities[1]. However, like the Coase Theorem, TU is a very strong assumption, one that can be undermined by a variety of frictions, such as transaction costs, social norms against certain forms of transfers,

[1]Though some still persist in search models – see Shimer and Smith (2001).

and commitment problems, among many others. By contrast, *nontransferable utility* (NTU) assumes that the division of the match surplus is exogenous. Generalized partially transferable utility settings are difficult to study, so papers in this literature typically focus on one or the other – TU for low friction markets and NTU for higher friction markets. In this paper, we'll assume NTU, focusing on matching markets like dating and marriage, where social norms limit transfers and the infrequency of negotiated dating and marriage contracts creates commitment problems. Some labor market applications include public sector jobs, where salaries are often non-negotiable, and entry level professional jobs that feature high levels of wage compression. Schools and students are another potential application. For simplicity, we focus on a market with symmetric distributions of agents across the two sides.

While we focus on a search model for tractability, these externalities are highly general. As illustrated in the first paragraph, they extend to frictionless environments like Gale and Shapley (1962). We decompose the externalities into to components: we call the first *intermatch externalities* and the second *intramatch externalities*. Intermatch externalities corresponds to the cost a matching agent imposes by taking their partner off the market. This can include dynamics such as thick market and congestion externalities. In our setting, this means that agents will match too aggressively on quality and too little on fit. This result is intuitive but not trivial: the fact that all agents agree on the ranking of the homogenous traits causes a strong intermatch effect – that is, if I am able to obtain the best potential match on the quality dimension, this means that I am removing that person from the market and thus preventing other agents from meeting that person. With more heterogeneous preferences, by contrast, my ideal match is less likely to be someone else's ideal match, so the intermatch effect is weaker or nonexistent. For example, if all agents' preferences are uncorrelated, pursuing your ideal match will not, in expectation, adversely affect the pool of potential matches for other matches. An intramatch externality arises when an agent's share of the surplus from a match isn't proportional to the overall match surplus. Pairings where one agent gets an unusually high payoff while the other gets an unusually low payoff may be rejected by the low payoff agent despite a good overall match surplus. In our model, this again leads to inefficiently aggressive matching on quality – an individual's match utility depends only on their match's type, while total match surplus depends on both, so matching to a Dud rather than a Stud is relatively more costly to the agent than to the planner. By contrast, fit payoffs are symmetric, creating no wedge between the agent's and the planner's incentives.[2]

Given these externalities, we proceed to reframe the matching market as a strategic platform operated by either a social planner or a profit-seeking monopolist and evaluate a variety of potential instruments to improve efficiency. In each case, we'll allow the monopolist to perfectly price discriminate with respect to Studs and Duds via fixed fees, so the monopolist's problem will reduce to the planner's problem of maximizing total surplus. We'll study three classes of instruments: pricing alone, splitting the platform, and censoring search. For pure pricing instruments and split platforms, we assume the platform can observe agent quality. For censored search we assume they can observe both fit and quality.

For pure pricing, we study two cases; the first is a two-part tariff. While per-interaction pricing[3] can resolve some externality issues, and in some special cases can achieve first-best, resolving the inefficient tradeoff between quality and fit requires the platform to make studs less picky when matching to duds and more picky when matching to studs, while per-interaction prices can only make an agent uniformly more picky or less picky. Second, we consider a more ambitious pricing scheme: match dependent pricing. If the platform can combine per-interaction pricing with pricing that depends both on own and match quality, the platform can generically achieve first-best. This can be framed as the platform implementing TU matching for its customers. However, this pricing scheme may be difficult to implement in practice.

For separate platforms, we show that, under CRM, creating separate platforms for Studs and Duds generates more surplus than a single platform. Externalities arise from matching on quality at the expense of fit, so partitioning agents by quality and forcing them to match solely on fit improves surplus. This parallels the separation result of Damiano and Li (2007). Examples of this include "elite" dating platforms like DateHarvardSQ and BeautifulPeople, as well as job search websites that target either professionals or low skilled workers. However, under LRM (and low search costs) a single platform can always outperform separate platforms. LRM implies that there are increasing returns to scale for platform size, and appropriate choice of acceptance regions will ensure that this effect dominates the externality cost.

Finally, we consider a novel instrument: censored search. Many matching platforms (Coffee

[2] Note that heterogeneous preferences do not require the symmetric payoffs we've assumed. However, symmetric payoffs are a logical consequence of horizontal preferences (if I'm close to you, you're also close to me). We briefly consider the case where the fit payoffs for the two matching agents are independent.

[3] A price charged for every draw an agent receives.

Meets Bagel, eHarmony, Chemistry.com) explicitly restrict their users' choice sets, only allowing them to see a small, curated subset of the market each day. One common justifications for this is lowering search costs by excluding partners you'd never match with. Another is that the platform acts as an expert middleman that is better able to assess match quality than you are. However, our analysis of matching externalities suggests a third explanation: some matches may be individually rational for their participants, but may generate negative externalities that make them unattractive to the platform. The platform can censor these draws to improve total surplus. Note that there is a clear limitation to this instrument because it can't directly induce agents to accept matches they don't want. By way of analogy, we can think of type I and type II errors in this setting – users commit a "type I error" when a match is rejected that ought to be accepted. A "type II error" occurs when a match is accepted when it should not be. Censored search can eliminate "type II errors", but not "type I"–at least not directly.[4] We show that, under some parameterizations of the model, censored search can improve total surplus and achieve first best, but in other cases there are no "type II errors" to correct and censored search has no marginal value for the platform. Our analysis of censored search relates to previous papers that have studied the potential benefits of restricting the cardinality of the choice set in matching problems (Halaburda et al. (2016)). Our work is distinct in that it focuses on censoring specific draws rather than the number or rate of draws overall. We conclude this analysis by studying censored search with per-interaction pricing. As mentioned above, censored search is limited by the fact that it can only address "type II errors". However, positive per-interaction costs act as additional search costs, forcing agents to be less selective. This brings a larger proportion of draws into the region where censored search has traction – matches that the agents will accept if presented to them. By making agents extremely unselective, all errors become "type II errors", and censored search can generically achieve first best.

Summarizing, we contribute to the literature by characterizing previously unstudied externalities that are endemic to matching problems with multi-dimensional preferences and evaluating a variety of instruments a platform can use to eliminate these inefficiencies. We also provide the first analysis of the value of censored search in eliminating externalities on a matching platform.

The outline of the paper proceeds as follows. Section 2 reviews the literature. Section 3

[4] Excluding "type II errors" changes the agent's optimization problem, which can lead to them accepting matches they would have rejected otherwise, but this second order effect is only effective in some cases.

provides the model setup. Section 4 studies the intermatch and intramatch externalities and provides two simulations that relax some of the assumptions made in the model. Section 5 provides a formal analysis of the efficiency properties of the six instruments discussed above, and Section 6 concludes.

2 Literature Review

This paper combines three literatures: search, matching, and pricing, The matching literature can be traced back to the seminal paper by Gale and Shapley (1962). Applications of matching theory to real-life markets have included the kidney exchange market (Roth, Sönmez, and Ünver (2005), Roth, Sönmez, and Ünver (2007), students to public school matching (Abdulkadiroğlu, Pathak, and Roth (2005), Abdulkadiroğlu et al. (2005)), and the job search literature (Bulow and Levin (2006)). In terms of utility specifications, the first two applications focus on non-transferable utility, which is similar to our paper. Bulow and Levin (2006), however, model transferable utility, where wages are taken as prices. Aside from the utility specification, another issue of interest when studying matching models is *assortation*. Positive assortation in equilibrium implies that high types match with other correspondingly high types, whereas negative assortation means that high types match with low types. Assuming both non-transferable utility and transferable utility, Becker (1973) proved conditions under which positive assortation occurs. As in many other matching papers, we also assume that matching is random, exogenous, and non-targeted.

Previous papers in the literature have focused on matching where agents have preferences over only one dimension. For example, in Burdett and Coles (1997), agents are vertically-differentiated with characteristics summarized into a single number, defined as agent's *pizazz*, which takes a value in the interval $[0, 1]$. Burdett and Coles analyze the matching equilibrium by studying steady-state conditions and find a rich set of equilibria. Agents match in assortative partitions in equilibrium, where the interval $[0, 1]$ for both types of agents is broken up into distinct pieces with agents in the same piece matching randomly with each other.

The search literature, on the other hand, started with the study of job search in the labor market (McCall (1970), Burdett (1978), Mortensen and Pissarides (1994), Rogerson, Shimer, and Wright (2005)). It has since expanded to include other applications as well, including the marriage and dating market (e.g. Cornelius (2003)). Applications of search theory to the dating market usually assumes non-transferable utility, as this mirrors real-life more closely than the assumption of transferable utility. In particular, Burdett and Wright (1998) and Adachi (2003) both study properties of equilibria in a search models with non-transferable utility.

This paper incorporates pricing theory in a world with externalities and a profit-maximizing monopolist. We study two cases: the first is when the monopolist can only operate one platform (perhaps due to high fixed or operating costs), and the second is when monopolist has the ability to operate two platforms that cater to the different types. This is in contrast to Rocher and Tirole (2010), who study platform competition with two-sided markets. Pricing in search markets has also been studied by other authors (e.g. Bester (1994)) in the context of a buyer-seller relationship where buyers are looking to buy a good from a price-posting seller. However, models such as Bester (1994) do not have the complexity of externalities which affect agent's optimal matching behavior in equilibrium, as in our model.

The closest paper related to the pricing framework in this paper Bloch and Ryder (2000), where they study the provision of matching services in a model of two-sided search. Agents are distributed on the unit interval and their utility is equal to the index of their mate. Bloch and Ryder (2000) find that in a search equilibrium, agents form subintervals and are only matches to agents inside their own class, a result that closely mirrors that of Burdett and Coles (1997). The two main differences between our paper and theirs is my inclusion of the heterogenous trait ψ and the fact that we assume that the monopolist utilizes a two-part tariff pricing structure. We are primarily interested in the signs of the per-interaction prices–be it positive, the non-distortionary (or, as we see later, the 'no externalities') level of 0, or even negative. Bloch and Ryder (2000), on the other hand, study how two separate pricing structures (uniform and commissions on the matching surplus) affect equilibrium behavior in agents. Another similar paper is Damiano and Li (2007), where the monopolist again faces two sides of the market with each side having characteristics distributed on a compact interval. The monopolist is able to choose a sorting and pricing structure in order to maximize revenue. They then show that the revenue-maximizing sorting is efficient.

We would like to make two final notes regarding the existing literature. To relate the horizontal (heterogenous) matching component used in our paper to the traditional horizontal-differentiation model presented by Salop (1979), note that the horizontal component utilized in our paper can be rationalized by preferences on the Salop circle. In particular, for any distribution function F

of agents on the circle, we can choose the cost of matching away from one's ideal type so that the idiosyncratic matching shock in our model is distributed according to F.

The second note relates to the following proposition:

Proposition 1 (Shapley and Shubik (1971))**.** *Suppose utility is transferable. A stable assignment must maximize total surplus over all possible assignments.*

Proposition 1 then tells us that in the case where utility is transferable, a *stable assignment*, which corresponds to the steady-state equilibrium in our model, must maximize total social surplus. In other words, in our environment with perfectly observable types and no bargaining, the monopolist charges prices to essentially reallocate surplus amongst agents *as though* utility is transferable. By maximizing total surplus, the monopolist in our model shifts the allocation of utility amongst agents from being non-transferable to utility being transferable.

3 Model

3.1 Environment

We study an infinite-horizon search model of matching, where each side of a two-sided market search for partners on the other side. Each side consists of two types of agents, characterized by a quality θ: High ("Studs") and Low ("Duds") (abbreviated H and L). Types are perfectly observable to the monopolist.[5] Time is continuous and both agents and the firm discount at the same rate r. We will be focusing on a steady-state equilibrium, which is defined by equating the total inflow into the platform with the total outflow out of the platform. Using these steady-state conditions on inflows and outflows allows us to solve for the prorportion of Studs on the platform, α, that makes the mass N of agents on the platform constant.

Once agents enter the market, they receive a stream of draws–that is, potential partners. This stream is characterized by a Poisson process with arrival rate λ. Throughout, we will make one of two assumptions about the rate of draws agents face: linear returns to matching (LRM) or constant returns to matching (CRM).[6]

[5]This is an assumption that is technically difficult to relax. For an exposition/explanations as to why this is the case, and some basic pricing results, please refer to this paper's online appendix at http://www.melatinungsari.com/research.html.

[6]Note that LRM is sometimes referred to as a quadratic search technology, owing to the quadratic nature of

Assumption 1 (Assumption 1A (LRM))**.** *Agents receive a rate of draws λ proportional to the mass of agents on the platform, normalized to N.*

Assumption 2 (Assumption 1B (CRM))**.** *Agents receive a constant rate of draws λ , normalized to* 1*.*

Linear returns to matching means that the frequency of draws is proportional to the mass of agents on the platform and that thick markets make search faster. Linear returns may be more realistic on online matching platforms like dating and job search sites where the choice set can easily and effectively be filtered down to a manageable size, effectively allowing users to search through potential matches faster when there are more agents on the platform. Constant returns to matching may be more appropriate for traditional forms of search where finding potential matches is time consuming and these frictions put an upper bound on the number of draws an agent can consider, regardless of the size of the market.

For each draw, agents incur a search cost of s, and meet a Stud with probability α and Dud with probability $1-\alpha$. This probability α is endogenous and will be determined in equilibrium by using steady-state conditions.

Once the agents meet, they learn each others' types, receive an idiosyncratic matching shock ψ (fit) and decide whether or not to match. If both agents decide to match, they do so forever. If one or both do not want to match, they return to search. The idiosyncratic shock ψ is drawn from a distribution on $[0, m]$ with cumulative distribution F and differentiable density function f such that $f(x) > \delta > 0$ for all x and some δ. Note that the two sides of the market have identical ψ distributions. A intuitive interpretation for ψ is given below.

The matching utility that an agent i receives when matching to agent j is given by

$$u_i(\psi_{ij}, \theta_j) = \theta_j + \psi_{ij} \tag{1}$$

We'll condense this to u_i where appropriate. There are two characteristics of the matching utility that we would like to elaborate on. First, note that agents receive the other agent's type as utility and not their own. This also means that all agents in the model prefer the Studs to Duds. Secondly, note that the shock ψ is the same for both partners, that is the shocks ψ that i

the total number of draws in the market as a function of the number of agents, and CRM is sometimes referred to as a linear search technology based on the linear rate of total draws in the market as a function of the total number of agents.

and j receive are *perfectly correlated*. We can interpret ψ as either a more general heterogenous preference or a horizontal preference. For example, the shock ψ may measure both agents' mutual 'fit' with one another. We argue that symmetric or highly correlated preferences are common to many heterogenous traits, and especially to horizontal traits. For example, suppose all agents prefer partners who are geographically closer. Then two agents who are close to one another will *both* find each other more attractive than agents who live far away. In any case, we will relax this assumption of perfect correlation of ψ's in simulations done in the next section, which is devoted to an in-depth explanation of the intermatch and intramatch externalities.

3.2 Agent's Problem

For all but a measure zero proportion of the time, agents are engaged in search and make no decisions. In the instant when an agent receives a draw, they simply decide whether to accept or to continue searching.

Definition 1. *Define a strategy for type i as*

$$S_i : \{\theta_L, \theta_H\} \times [0, m] \ \rightarrow \{accept, reject\}$$

The interpretation for the strategy is that, for a given own-type, it takes the ψ-draw and type of the opposite agent as arguments and returns a decision on whether or not to form a match. Then $\overrightarrow{S}$ is the strategy profile for the continuum of agents on both sides. Define

$$o_i(S) \equiv P[match|draw, i, S]$$

We'll often truncate this to o_i or o_{θ_i}. Define $U_{\mathrm{i}}(s)$ as the expected lifetime utility for i while searching, which we'll also truncate to U_{θ_i} or U_i as appropriate. Define M_i as the set of potential partners that will generate matchs (mutual acceptance) for i. The dynamic program for this environment gives us the following condition for a small time period dt:

$$U_{\mathrm{i}} = \frac{U_{\mathrm{i}}(1 - \lambda o_i dt) + \lambda o_i E_{M_i}[u_i]dt - s\lambda dt}{1 + r\, dt}$$

Taking the limit as $dt \rightarrow 0$ and rearrainging, we have

$$U_{\mathrm{i}} = \frac{o_i E_{M_i}[u_i] - s}{r/\lambda + o_i} \tag{2}$$

Then, at any time t where agent i has received a draw, the continuation value of rejecting that draw is precisely $C_i \equiv U_\text{i}$.

Optimality requires that an agent accept any draw whose utility exceeds U_i. Note that (2) is independent of the current draw, while u_i is strictly increasing in ψ_{ij} and θ_j. This implies that i's optimal strategy takes the form of a *threshold strategy*.

Definition 2. *Agents optimally accept a draw when the match utility is at or above continuation value*[7]*, and choose to continue searching otherwise. Formally,*

$$S_i^*(\theta_j, \psi_{ij}) = \begin{cases} \textit{reject} & \textit{if } \psi > \psi_i^*(\theta_j) \\ \textit{accept} & \textit{if } \psi \leq \psi_i^*(\theta_j) \end{cases} \tag{3}$$

There are then two thresholds for each agent of quality θ, $\psi_\theta^* = \{\psi_\theta^*(H), \psi_\theta^*(L)\}$. The optimal thresholds satisfy the following equation:

$$\theta_H + \psi_\theta^*(H) = \theta_L + \psi_\theta^*(L) = C_\theta \tag{4}$$

It immediately follows that

Lemma 1. *For any $\theta \in \{\theta_L, \theta_H\}$, $\psi_\theta^*(L) \geq \psi_\theta^*(H)$.*

This proposition informs us that any fixed type of an agent is pickier about dating duds than studs. This is a direct consequence of studs being seen as universally more attractive. Now, we characterize the agent's optimal thresholds and expected continuation utilities. This will allow us to rank the relative positions of the four thresholds. In particular, as we will soon see, the Studs will be *pickier* when it comes to matching. An agent i is said to be *pickier* in the matching process than agent j if $C_i > C_j$.

Proposition 2. *In a steady-state equilibrium, $C_H \geq C_L$.*

Proof. High types have a strictly higher quality and are otherwise identical. Thus, in equilibrium they must be accepted whenever a low type would be accepted and, for any feasible low type strategy, they can replicate the expected payoffs by accepting every pairing that would be mutually accepted for the low type and reject all other draws. Thus, C_L is a lower bound for the high type's continuation value. □

[7] We will exclude mixed acceptance strategies in this paper.

Corollary 1. *For all* $\theta \in \{\theta_L, \theta_H\}$, $\psi_H^*(\theta) \geq \psi_L^*(\theta)$

From the above propositions, we can deduce that H always has the upper hand in determining whether or not a match forms when two agents meet. This means that we can reduce the set of thresholds to characterize. In particular, a Dud's threshold for matching with a Stud ($\psi_L^*(H)$) is a non-binding constraint since the decision whether or not to marry depends solely on $\psi_H^*(L)$. Thus, we now only have to characterize three thresholds: $\psi_H^*(H), \psi_H^*(L), \psi_L^*(L)$.

With these continuation values in hand, we can express 2 explicitly for Studs and Duds:

$$C_H = \frac{\left(\alpha \int_{\psi_H^*(H)}^a (\theta_H + \psi)\, f(\psi)\, d\psi + (1-\alpha) \int_{\psi_H^*(L)}^a (\theta_L + \psi)\, f(\psi)\, d\psi\right) - s}{r/\lambda + o_H} \tag{5}$$

where

$$C_L = \frac{\left(\alpha \int_{\psi_H^*(L)}^a (\theta_H + \psi)\, f(\psi)\, d\psi + (1-\alpha) \int_{\psi_L^*(L)}^a (\theta_L + \psi)\, f(\psi)\, d\psi\right) - s}{r/\lambda + o_L} \tag{6}$$

where

$$o_H = \alpha \left(1 - F\left(\psi_H^*(H)\right)\right) + (1-\alpha)\left(1 - F\left(\psi_H^*(L)\right)\right)$$

and

$$o_L = \alpha \left(1 - F\left(\psi_H^*(L)\right)\right) + (1-\alpha)\left(1 - F\left(\psi_L^*(L)\right)\right)$$

Note that the following proposition holds α and λ fixed. In equilibrium, however, α and λ are not fixed and are determined endogenously. This is because of the previously mentioned fixed point problem: α and λ affects the thresholds, which then in turn affect α and λ. However, it is useful to have the comparative statics by holding them constant, since the agents themselves take them as constant when they optimize.

Proposition 3. *Fix* α *and* λ. *For distributions with continuous support, the following are true:*

1. *All thresholds strictly decrease with* s.

2. *All thresholds strictly decrease with* r.

3. $\frac{\partial C_H}{\partial \alpha} > 0$

4. $\frac{\partial \psi_H^*(H)}{\partial \alpha} > 0$, $\frac{\partial \psi_H^*(L)}{\partial \alpha} > 0$

Proof. Appendix. □

The first result is that thresholds strictly decrease with search costs. This means that higher search costs cause agents to become less picky since the cost of searching on the platform has now increased. A lower r, which corresponds to more patient agents, causes agents to become less picky. All else equal, agents are now content with staying on the platform and waiting for longer to obtain a better match and random draw ψ. The last two results of the previous proposition relate to the behavior of the studs when the proportion of the studs, α, changes. In particular, the results show that studs are both better off and pickier when there are more studs on the platform. This is not a surprising result since studs are seen as the superior type by everyone on the platform. We are not able to prove similar results relating to how the optimal strategy of the duds changes with respect to changes in the proportion of studs on the platform. This is because this result would depend strongly on whether or not studs are willing to match to duds. In particular, one can imagine a situation where studs are significantly better than duds (i.e. $\theta_H - \theta_L$ is very large), in which case studs would never match to duds. This situation would cause studs to be much worse off in the presence of many duds. In particular, the value of the platform as a means to obtain a match decreases to the duds–he is not able to increase his odds of obtaining a match by joining a platform in which the vast majority of others on the platform do not wish to match to him.

3.3 Dynamics

We will be focusing on steady-state equilibrium in this paper. To do so, we introduce the dynamics in the model. At time t, there is a time-invariant exogenous inflow rate, normalized to 1, of studs joining the platform. The exogenous inflow rate of duds is $i_L \in R$.[8] There is also a mass of agents of each type already on the platform, denoted N_t. For each quality, agents leave based on the outflow rate for that quality, the mass of agents of that quality $N_t\alpha_t$ and $N_t(1-\alpha_t)$, respectively, and the rate of draws. We can then state the transitional equation for N_t, which is the following

$$N_{t+dt} = N_t = N_t + \, 1\, dt + i_L\, dt - N_t\lambda_t\alpha_t\, o_{tH}\, dt - N_t\lambda_t(1-\alpha_t)\, o_{Lt}\, dt$$

Rearranging, taking the limit, and assuming a constant N_t, we have

$$N = \frac{i_L + 1}{\lambda\left(\alpha o_H + (1-\alpha)o_L\right)} \tag{7}$$

[8]When unambiguous, we will abuse notation by using i to represent inflow in a single quality case of the model.

Now, we are ready to define a steady-state. The steady-state will determine the values of α and λ.

Definition 3. *A steady state consists of s, outflows, and a constant distributions of agents* $\{N, \alpha\}$ *on the platform. It is characterized by the following equations:*

$$1 = o_H \lambda \alpha N \tag{8}$$

$$i_L = o_L \lambda (1-\alpha) N \tag{9}$$

$$N = \frac{i_L + 1}{\lambda \left(\alpha o_H + (1-\alpha) o_L\right)} \tag{10}$$

Given this definition, the proportion of H types on the platform, α, can then be solved for from the steady-state equations.

$$\alpha = \frac{o_L}{o_L + o_H i_L} \tag{11}$$

There are some intuitive properties of α that are worth highlighting. First of all, note that α becomes smaller with a higher o_L. As duds types leave the platform faster, the proportion of studs types on the platform will increase. Similarly, the quicker studs leave the platform, the lower α becomes since o_H has increased. This analysis can also be conducted on the proportion of agents that are let into the platform: as i_L increases, α decreases; and as more studs enter, the higher the proportion of studs on the platform becomes.

Given this expression for α, we can complete our analysis of λ and N. Plugging 11 into 7, we have

$$N = \frac{\frac{1}{o_H} + \frac{i_L}{o_L}}{\lambda}.$$

For CRM we have $\lambda = 1$ and

$$N = \frac{1}{o_H} + \frac{i_L}{o_L} \tag{12}$$

For LRM we have$\lambda = N$ and

$$N = \sqrt{\frac{1}{o_H} + \frac{i_L}{o_L}} \tag{13}$$

4 Multidimensional Matching Externalities

We will now discuss the externalities studied in this paper in detail. For expositional ease, we will first consider a frictionless model in the vein of Gale and Shapley (1962). Before we do so, we would like to introduce two important concepts relating to matching markets, which are the *matching function* or *assignment* and *stability*.

Definition 4 (Matching μ)**.** *A matching μ is a mapping from each agent to their match.*

Definition 5 (Stability - Transferable and Non-Transferable Utility)**.** *In the case where utility is non-transferable:*

- *A matching μ is stable if there is no a and b such that $b \succ_a \mu(a)$ and $a \succ_b \mu(b)$. (a, b) is called a blocking pair.*

In the case where utility is transferable:

- *A matching μ is stable if $\exists$ a feasible allocation rule $v : A \cup B \rightarrow \mathbb{R}$ giving the payoff for each matched agent such that there is no a and b such that $u(a, b) > v(a) + v(b)$. (a, b) is called a blocking pair.*

As mentioned before, the intermatch and intramatch externalities are not specific to search models, but generally present in non-transferable matching models even without search costs. Consider a matching market identical to the one described above, but without search costs and with a finite and equal set of agents on each side. Agents can observe the type of every possible match and costlessly propose to potential suitors, so there will be a stable matching where every agent matches to the most preferred partner that will accept her (him). Clearly, one's matching decision will affect others, since it changes the set of available partners for all other agents. In particular (and here, we introduce some new notation), the man m that a woman w receives must be withheld from some other woman w', who counterfactually would have received m as a match if w had matched differently.

However, self interested agents do not care about what happens in other matches. Shapley and Shubik (1971) showed that, with perfectly transferable utility, transfers can allow such an agent w' to force others to internalize the costs they impose on her in a manner analogous to the Coase Theorem. However, with non-transferable utility there is no mechanism available to internalize externalities. Note also that agents in a multidimensional matching environment will generically have to make tradeoffs between the various traits they care about. Unless every agent can match to the most desirable possible partner along all traits simultaneously, they will have to choose between matches that are better along one dimension and matches that are better along another.

Thus, if agents have preferences over a vertical trait θ, where all agree on the preference ordering; and a heterogeneous trait ψ, where every agent's preference ordering is independent and identically distributed, we can expect agents without access to transfers to make an inefficient trade-off between the traits. This is because, along the vertical trait, a pursuing a good match for oneself must mean a bad match for another—there are only so many high type agents, and this intermatch externality ensures that taking one out of the market worsens the outcome for at least one other agent along this trait. With the heterogeneous trait, by contrast, pursuing a good match for oneself has no effect on the distribution of remaining agents in expectation. Thus, good θ matches impose a negative externality on some other agent, while good ψ matches do not, and self interested agents will match too aggressively on θ.

With modular preferences over θ, or, more generally, vertical traits, this effect is especially stark. In a model where one's utility from a match is their match's type (or more generally where match utility is the sum of the agent's types) the total surplus is invariant to the matching assignment—it doesn't matter who matches to whom, simple algebra shows that the total surplus in the market is the sum of every agents type, with different assignments simply changing the order of summation. In a model with a modular vertical trait and another trait, as in this paper, total surplus must then depend only on sorting along the other trait, since the surplus accruing from the vertical trait is invariant to assignment.

Formally, consider a frictionless two-sided matching market with

- n agents on each side.
- k modular vertical traits $\theta = (\theta_1, ..., \theta_k)$, where the vector for vertical characteristics of a a male agent (m) i is θ_i^m, and his jth vertical characteristics is $\theta_{i,j}^m$.
- l other traits $\psi = (\psi_1, ..., \psi_L)$.

- match surplus for man i and i's match $\mu(i)$ given by

$$u(\theta_i^m, \theta_{\mu(i)}^w, \psi_i^m, \psi_{\mu(i)}^w) = \sum_{j=1}^{k} (\theta_{i,j}^m + \theta_{\mu(i),j}^w) + f(\psi_i^m, \psi_{\mu(i)}^w)$$

Define TSS_θ as the total social surplus from matching on the vertical type θ, TSS_ψ as the total social surplus from matching on heterogenous traits ψ, and TSS as the overall total surplus (i.e. $\text{TSS} = \text{TSS}_\theta + \text{TSS}_\psi$). Then, we have

$$\text{TSS}_\theta \equiv \sum_{i=1}^{n} (\sum_{j=1}^{k} (\theta_{i,j}^m + \theta_{\mu(i),j}^w))$$

$$\text{TSS}_\psi \equiv \sum_{i=1}^{n} f(\psi_i^m, \psi_{\mu(i)}^w)$$

$$\text{TSS} = \text{TSS}_\theta + \text{TSS}_\psi$$

Proposition 4. *In the above environment, a non-transferable utility matching must exhibit weakly lower TSS_ψ than the TSS_θ-maximizing assignment, which is also the transferable utility stable matching.*

Proof. Appendix. □

This only shows that the non-transferable matching must have a weakly lower TSS due to ψ traits, but generally, in a non-transferable utility framework, agents will prefer to get better θ draws, even if the overall θ endowment is unchanged, and will thus make inefficient tradeoffs against ψ matching, so the inequality will typically be strict. If vertical payoffs are supermodular, it will often be that the TSS_θ will be higher for non-transferability than for the first best assignment.

This externality can be translated into the search environment as well, but the mechanism of action is a bit more involved. Without frictions, we can talk of specific agents preventing specific other agents from getting a desired match, making the externality extremely clear. In a search model with a continuum of agents, by contrast, we can only talk about measurable masses of agents, distributions of potential draws, and expectations over matching outcomes. However, in a steady-state search model, inflow distributions must equal outflow distributions, and thus the distribution of matches that agents receive each period must equal the inflow distribution. Thus, if a nonzero mass of women match to high type men, that equal mass of high quality men is unavailable to other women, and they must receive lower quality matches on average than if they were able to match to this group. Again, a vertical trait induces a clear externality, while a heterogeneous trait generally does not. The mechanism by which this assignment happens is search, however, so counterfactually different assignments must be implemented through differing distributions of agents and cutoff strategies—that is, this externality is mediated by the familiar thick market and congestion externalities. For example, if high types only accept one another, this will change the distribution of agents on the platform, and that will change the distribution of draws agents face and the set of agents who will accept them. Thus, the intermatch externality corresponds to the effect of agent threshold strategies on α and λ in our search model.

The other externality we study is the intramatch externality. This sort of externality appears in many economic environments, notably in Coasian models. Fundamentally, it arises from a wedge between private match utility u_s and match surplus u. If u_s is not proportional to u, agents value a given match differently from a social planner, and may make socially inefficient acceptance and rejection decisions. This externality can appear (along with the intermatch externality) in a one-dimensional model due to a tradeoff between time (discounting or search costs) and match quality, and is in fact present in models like Burdett and Coles (1997), though it has generally not been discussed explicitly in the literature. A simple example is the case with utility being your match's type and non-transferability. Total surplus may be quite high when a high type matches to a low type, and it may be socially optimal for the match to proceed so as to avoid more time costs, but the high type only receives the low type utility, ignoring the large benefit she provides to her partner, and may choose to reject. Because there are no transfers, the low type cannot offer some on their large benefit in order to induce a match. In multidimensional models there is an additional tradeoff between traits, and thus this externality can appear even in environments without time costs.

In our model, there is a wedge between agents' private matching utilities (u) and the resulting match surplus (u_s) for vertical traits, due to the assumption that utility obtained by an agent is his match's type, but not with the heterogeneous trait ψ, which is symmetric across the sides of the market—that is, your ψ draw is the same as your match's ψ draw, so u and u_s due to ψ are proportional. This means that both externalities work in the same direction—people match too aggressively on the vertical trait. Ideally, we would break the model out into cases with each externality in order to decompose the effects, and that is an avenue for future work, but we will argue to this coincidence of externalities fits with the stylized facts of multidimensional matching markets.[9]

Specifically, many traits over which agents have heterogeneous preferences will tend to exhibit symmetric payoffs and thus no or small wedges. As mentioned before, horizontal traits are a common type of heterogeneous preference, and they have symmetric payoffs by construction—if two agents m and w prefer matches closer to one another, then if m gets a high payoff from w along this dimension they must be close, which means that w also gets a high payoff from m. This can also apply to traits like race, religion, values, or even traits that factor into decision making like patience and risk aversion where agents often prefer a match similar to themselves.[10] Even with heterogeneous traits that are not horizontal, symmetric payoffs are quite plausible—traits like mutual chemistry are likely to be approximately symmetric. With vertical traits, by contrast, symmetry is possible but requires strong functional form assumptions that are orthogonal to the homogeneous preference ordering assumption. Thus, a model with a wedge only for the vertical trait captures these arguments in a simple and stylized way. In our search model the intramatch externality corresponds to the effect of ψ_{HL} on duds.

We will now explore several simulations of a frictionless non-transferable utility market analogous to the baseline model of this paper (without a strategic platform), as well as several permutations of this model. The results of the simulation are in Table 1. We also estimate the corresponding transferable utility assignment, again using the Shapley and Shubik (1971) result that transferable utility gives the TSS-maximizing assignment. This allows us to see the first best outcome and evaluate the externalities caused by non-transferable utility. We include these simulations both to show that these externalities are extremely general and do not depend on the assumption of large search frictions, as well as to explore the effect of alternate assumptions on match surplus function on the externalities in this environment.

These Monte Carlo simulations study markets with 50 agents on each side, and utilize 40 iterations each. A man m and a woman w obtain the following utility when deciding to match:

$$u_m = \frac{(1+a)\theta_m^a \theta_w + \psi_m}{2}, \; u_w = \frac{(1+a)\theta_w^a \theta_m + \psi_w}{2}$$

Note that a is a parameter determining the supermodularity of the utility function. When a is

[9] We will also study frictionless analogues with different assumptions on which traits induce wedges shortly.

[10] You may prefer a match with similar patience and risk aversion since you'll be making many joint decisions and you'd prefer concordance.

zero, the utility function is analogous to that of the baseline model, while $a = 1$ gives a vertical payoff component comprised of the product of one's own type and one's match's type. This means that higher types generate more utility, and thus make matching high types to high types and low types to low types generates more surplus than matching high and low types, all else equal. We'll treat the case where $\psi_m = \psi_w$, as in the baseline model, and also treat the case when they are independently drawn, dropping the symmetry assumption. Note that, with supermodular utility, the vertical trait in fact exhibits symmetric payoffs, so we will be able to model all four permutations of wedge and no-wedge over the two traits. ψ and θ are drawn from $U[0, 1]$, and the $(1 + a)$ term is a normalizing factor to account for the fact that, with the supermodular specification, utilities will generally be lower since types are drawn from $[0, 1]$ with an average of 0.5, so when $a = 0$, $\theta_m^a \theta_w$ is 0.5 on average, while when $a = 1$, the match's type is now multiplied by another draw from $[0, 1]$, yielding 0.25 in expectation with random assignment. Defining an agent's own type as (θ, ψ), match's type as (θ', ψ'), and a match μ as $\mu(\theta, \psi) = (\theta', \psi')$, where $\mu_1(\theta, psi) = \theta'$ and $\mu_2(\theta, \psi) = \psi'$. With vertical traits, we are interested in how aggressive the matching is along this trait, since we have predicted that it will be too high with non-transferable utility. Since high types are mutually desirable and low types the opposite, we can expect sorting along the vertical trait to induce assortative matching as per Becker (1973), with high types using their attractiveness to ensure a high type match and low types being stuck with other low types. Thus, more effort to match along the vertical trait should induce more correlation between own θ and match's θ, and in a sufficiently large market with only vertical sorting, $E(\mu_1(\theta, \psi)) = \theta$. Thus we will specify a simple regression

$$\mu_1 = \beta\theta + \epsilon$$

and report the β and R^2 vales. Additionally, we will report TSS, as well as TSS_θ and TSS_ψ, with the expectation that TSS_ψ and TSS should be higher for transferable utility and TSS_θ should be weakly higher for non-transferable utility.

Comparing non-transferable utility and transferable utility when $a = 0$ and ψs are correlated, the baseline environment for the paper, we see that TSS_θ is the same for both, as guaranteed by the above proposition—with modular utility, the matching assignment doesn't matter. However, in non-transferable utility agents sort quite a bit on the vertical trait—high types get better matches, but those better matches come at a cost to low types. The coefficient is relatively close to 1 and one's own vertical type explains 60% of the variation in match's vertical type. This comes at a clear cost to TSS_ψ, which is much lower for the non-transferable utility case, and thus TSS is also much lower, demonstrating the cost of these externalities.

Comparing non-transferable utility and transferable utility when $a = 1$ and ψs are correlated, we see sorting on the vertical trait for both the first best and the non-transferable utility assignments since supermodularity means that some assortation is desirable, but there is more sorting in theta with non-transferable utility and worse assignments in ψ in terms of TSS_ψ. Now there is no wedge for either trait, so these externalities come from intermatch, and intermatch is also weaker since the supermodularity means the costs imposed on low types are scaled by their own type and are thus lower than the benefits accruing to high types. However, there is still a modest loss in TSS due to intermatch.

Comparing non-transferable utility and transferable utility when $a = 0$ and ψs are uncorrelated, we again see sorting on the vertical trait only for non-transferable utility assignments. Now there is a wedge for both traits, so the intramatch externalities should largely cancel. We see a large shortfall in TSS_ψ due to intermatch with non-transferable utility, demonstrating the significance of this externality, especially with modular or close to modular utility.

Finally, comparing non-transferable utility and transferable utility when $a = 1$ and ψs are uncorrelated, we again see sorting on the vertical trait for both assignments. Now there is a wedge only for ψ, so the externalities have opposite effects. The intermatch externality clearly dominates here, since there is still more sorting along the vertical trait in the non-transferable utility case and less along ψ.

5 Resolving Multidimensional Matching Externalities

Having established the ubiquity of these externalities in settings without transfers, we now ask what instruments a platform could utilize to counteract them. We look both at pricing strategies, like two-part tariffs, and changes to the structure of the market itself, such as partitioning the platform along quality. In each subsection, we derive some basic results about the efficiency of the instrument, then provide some qualitative discussion of its implementability. Note in each case that expected utility is constant across types and own type does not directly enter into the utility function, so it is without loss of generality for us to ignore misreporting issues –incentive compatibility constraints will be satisfied for every agent in all of the following specifications. Before that, however, we'll establish a few preliminary results. FIrst, we show that maximizing total surplus is possible.

Lemma 2. *Total surplus has a well defined maximum in the space of cutoffs* $\Psi \equiv \{\psi \in [0, m]^4\}$.

Next, we formalize the fact that ψ_{LH} is irrelevant.

Lemma 3. *Any* ψ *generates an identical set of accepted matches to some* ψ' *of the form* $\{\psi_{HH}, \psi'_{HL}, \psi'_{HL}, \psi_{LL}\}$.

Proof. Appendix. □

Finally, show that search costs are irrelevant to optimality in censored search.

Lemma 4. *First best* ψ^* *with censored search is invariant to search costs.*

Proof. Appendix. □

5.1 Pricing

We'll first consider the exclusive use of price instruments, starting with the standard two-part tariffs and moving on to more complex pricing strategies.

For two-part tariffs, we first present the non-distortionary pricing model in which there are no externalities. In particular, agents are identical. In the second subsection, we study the cases where there are two types of vertically-differentiated agents. In the third subsection, we focus on three cases for the idiosyncratic matching shock ψ: first, the case with no fit shocks; second, the special case where there are small idiosyncratic shocks; and third, the case where ψ is arbitrarily distributed on $[0, m]$. All cases considered in this section assume agents' types are perfectly observable by the monopolist.

The assumption of perfect observability may not hold perfectly in reality; this being said, it allows for tractability. This assumption can be relaxed and we refer the reader to a related paper focused on the case of unobservability.[11]

[11] This may be found here

5.1.1 Two-Part Tariffs in a Model With No Externalities

Consider the baseline model where there is only one vertical type, perfectly correlated matching shocks ψ, and CRM. This model with no externalities provides a baseline for the more complicated model later on, where externalities are introduced through the α term and the tradeoff between choosing to match more aggressively on the homogenous versus heterogeneous dimension. To solve for the steady-state equilibrium, note that the monopolist acts a social planner who maximizes total surplus in the economy. Now, note that the only way prices affect the agent's problem in the case with no externalities is through the agent's threshold ψ^*. For ease of exposition, since utility is additively separable, write the expected discounted utility of agents as

$$EU(\psi^*) \equiv U(\psi^*) - \sigma - p\, D(\psi^*)$$

where U is the expected discounted match surplus before prices, $\frac{\int_{\psi*}^{m}(\theta+\psi)f(\psi)d\psi - s}{r+(1-F(\psi^*))}$, D is the time discounted expected number of dates the agent goes on before leaving the platform, $\frac{1}{r+(1-F(\psi^*))}$, p is the per-interaction price charged by the platform per draw, and σ is the fixed price. Given prices, agents will optimize according to the first-order condition with respect to ψ^*, yielding

$$\frac{\partial EU(\psi^*)}{\partial \psi*} = 0 \implies \frac{\partial U(\psi^*)}{\partial \psi^*} = p\frac{\partial D(\psi*)}{\partial \psi*}$$

However, without externalities, the platform should maximize individual utility before prices, $U(\psi^*)$, so the optimal p must be zero. Now, note that the monopolist profit function is given by

$$\Pi = 2\{i\sigma + \frac{ip}{r+o}\} \tag{14}$$

In most of this paper, we'll take inflows as given, but one might ask whether it is optimal to allow all agents on the platform. The profit function linearly increases in i, which implies that the monopolist optimally sets $i = 1$.[12] This fully characterizes the solution to the monopolist's optimization problem for the case where there are no externalities.

This provides a benchmark for the case where there are no distortions caused by externalities imposed by different types of agents on each other. In particular, the firm in acting as a social

[12] The profit function takes into account the cohort that entered the platform in steady state and follows them throughout the entirety of their lives. This is in contrast to a similar (but incorrect way) of calculating profit, which is by calculating the sum of profits that is able to be extracted from the mass of agents *in* steady-state, regardless of the when they entered the platform.

planner does not have to correct the matching behavior of agents on the platform by charging a non-zero per-interaction price since agent's optimality conditions are already aligned with those of society.

5.1.2 Two-Part Tariffs in a Model With Externalities

Consider now a model where there are two types of agents, namely the studs (H types) and the duds (L types). We will analyze the case in which the firm is not able to separate the two types of agents into distinct platforms. The firm may not be able to separate the two types of agents for to a variety of reasons. For example, an online dating firm may specialize and operate only a single website, focusing on aggressively advertising and managing the website. The firm may also face high fixed and operating costs to operate more than one website, or may be faced with legal constraints that prevent them from segregating the two types.

Before proceeding to the analysis, we will first outline the monopolist's problem.

Monopolist's Problem

We assume that the monopolist *fully commits* to a constant path of prices throughout time, charging $\{p^* = (p_H^*, p_L^*), \sigma^* = (\sigma_H^*, \sigma_L^*)\}$ at each t.

Definition 6 (Monopolist Profit)**.** *The firm's expected profit in steady-state is*

$$\Pi_{ss} \equiv 2\left\{i_H\sigma_H + i_L\sigma_L + \frac{i_L p_H}{r/\lambda + o_H} + \frac{i_L p_L}{r/\lambda + o_L}\right\}$$

This function can be thought of as the profit obtained by following the group of agents of type H and L that entered in the steady-state until the end of their time on the platform.

Given that types are perfectly observable, the monopolist is only concerned about the *individual rationality* (participation) constraints. The optimization problem for the monopolist is defined as follows.

$$\max_{p_H, p_L, \sigma_H, \sigma_L} \Pi_{\text{ss}}(\alpha(p_H, p_L), p_H, p_L)$$

$$\text{s.t.} \quad (IR_H) \quad C_H(\alpha(p_H, p_L), p_H) - f_H \geq 0$$

$$(IR_L) \quad C_L(\alpha(p_H, p_L), p_H, p_L) - f_L \geq 0$$

subject to the steady-state conditions in Definition 3.

The pricing problem is complicated by the fact that changes in prices affect not only one group of agents, but also the interactions between the group and the other through the steady-state α. However, since types are observable, the firm will extract all surplus from the agents–if an IR constraint doesn't bind, the firm can raise the corresponding fixed fee without changing agent behavior, increasing profit. This implies that the pricing problem is equivalent to the planning problem in which the firm maximizes the total surplus of the agents on the platform, only to charge prices to extract it all from them.

No to Small Idiosyncratic Matching Shocks

Suppose, to begin, that there are no idiosyncratic shocks in this model. An agent of type i obtains the following utility when she chooses to match with agent j:

$$u_i(j) = \theta_j$$

An agent obtains utility equal to the type of her match. Agents are risk-neutral and share the same discount rate with the firm. We can think of there being two steady-state equilibria: a *pooling* equilibrium where H and L types match not only amongst themselves but also with each other, and a *separating* equilibrium with H types only match with other H types, and L types only with other L types.

Since types are perfectly observable, the profit-maximizing monopolist will extract all surplus from the agents by using a combination of per-interaction prices $p = (p_H, p_L)$ and fixed fees $\sigma = (\sigma_H, \sigma_L)$. All agents have the same outside option of obtaining zero utility when not on the platform. The individual rationality constraints for each type will bind, giving

$$C_H - \sigma_H = 0$$

$$C_L - \sigma_L = 0$$

An agent's strategy under this setup is either to accept all matches or to only accept H types. In equilibrium, since H determines whether or not a match forms, L's only strategy will be whether or not she will match with another L.

Under this setup, the total social surplus will be a function of all the types of agents on the platform. For concreteness, consider a frictionless analogue with 4 agents in the market, with types H, H, L, L. There are only two cases—the first in which $H - H$ match and $L - L$ match, and the other where two $H-L$ matches form. It is clear then that the total surplus that may be extracted by the monopolist is the same for both cases; this implies that the monopolist does not care about who matches with whom, but only about the total surplus that may be extracted. This result obtains from the fact that the matching utility is additively separable (modular). Another important fact to note is that an agent's contribution to the total social surplus is his or her own type. This is due to the fact that since an agent obtains his or her partner's type in a marriage, each agent then contributes his or her own type to the surplus.

We can now define the total social surplus on the platform. This is given by

$$\text{Total Social Surplus (TSS)} = i_H \frac{o_H \theta_H - s}{r/\lambda + o_H} + i_L \frac{o_L \theta_L - s}{r/\lambda + o_L} \tag{15}$$

Note that prices are implicit in the expression of total social surplus. We can ignore explicit expressions of prices since any given expected discounted lifetime utility of being on the platform can be *fully* extracted by the monopolist given a combination of prices. The prices only enter in the expression of total social surplus in the outflows o_H and o_L, where higher prices would cause agents to be less picky and so, leave the platform more quickly.

In the following proposition, we prove a strong and interesting result about this model, namely that given the setup and *any* range of parameters, the monopolist will price per-interaction prices to extract all surplus from the agents and make them leave *at the first draw.* Also, given any range of parameters, the monopolist will choose to let all the L types in, regardless of the value of θ_L.

Proposition 5. *Consider the model with no idiosyncratic matching shocks. In the steady-state equilibrium, the profit-maximizing firm will*

1. *Set* $i_L^* = i_L$.
2. *Charge* p_H, p_L *sufficiently high to induce all agents to leave at the first draw they receive on the platform.*

Proof. Appendix. □

There is an intuitive interpretation for this result in terms of the discount rate r. Consider two cases for the discount rate: $r = 0$, and $r > 0$. In the case where $r = 0$, any pricing strategy that the firm sets will give the same TSS, since there is no discounting. In the case where $r > 0$, note that TSS is strictly decreasing in the the time that the agents stay on the platform. This being, the pricing strategy that forces everyone to leave as soon as possible gives a higher TSS than any other pricing strategy. This is direct for the CRM case, and, while under LRM slower exit increases the arrival rate, creating a countervailing increase in matching, this effect is bounded below the direct effect of slower exit. Thus, the optimal pricing strategy for the firm is to set per-interaction prices sufficiently high to induce all agents to leave in the first period.

Now, we would like to make strides towards understanding the model where the idiosyncratic shock is sufficiently small. We have already shown what the steady-state equilibrium looks like in the context of no idiosyncratic shocks. What happens when these shocks are small, i.e. when the *support* of the distribution of ψ is small? It turns out that the result for when the shocks are small relative to the size of the types (θ_H, θ_L) is the same as in the case where there are no shocks for a range of parameters[13]. A sufficient but not necessary condition for the shock ψ being small is $\frac{\theta_L}{m} \geq r$.

Proposition 6. *Suppose $\psi \in [0, m]$ and CRM. Then for parameterizations such that $\frac{\theta_L}{m} \geq r$, the profit-maximizing monopolist*

1. *Sets $i_L^* = i_L$.*
2. *Charge p_H, p_L sufficiently high to induce all agents to leave at the first draw they receive on the platform.*

Proof. Appendix. □

The intuition for these results is as follows. The monopolist only cares about the total amount of social surplus in the economy. The total social surplus declines as time goes on and agents are still on the platform based on the discount rate r. This being, the monopolist will want to force agents to match immediately while on the platform *unless* there is a non-zero probability of agents obtaining a sufficiently high draw ψ, which will improve their utility and so increase the amount of surplus that can be extracted from them. In the case where the idiosyncratic matching shock ψ is small relative to the size of the types θ_H and θ_L, the incentives for the monopolist to let agents stay on the platform for long are corresponding small. In particular, with both agents and the monopolist valuing the future very little (implying high values of r), there is no incentive

[13]We do not treat the LRM case due to a lack of tractability

for either side to stay longer on the platform when the benefit of waiting on the platform for a better ψ is small.

One natural question to ask now is whether the firm charges different per-interaction and fixed prices to the studs and duds under the parameter range where the pricing strategy of charging sufficiently high prices to induce agents to match immediately is implemented. What is observed is that since the monopolist firm lets all agents in to the platform and outflow rates for both studs and duds are exactly 1, the probability of meeting a stud or dud is exactly (denoted α) is also exactly $\frac{1}{2}$. Since the chance of meeting a stud is the same for both types of agents, and per-interaction prices are set to induce both agents' acceptance thresholds to be 0 (agents marry their first match), agents expected lifetime utility of being on the platform is the same regardless of type. Thus, we obtain the following result that the profit-maximizing monopolist charges the same fixed prices to both H and L types, despite the fact that types are observable.

Proposition 7. *Suppose $\psi \in [0, m]$, CRM, and $\frac{\theta_L}{m} \geq r$. In the steady-state equilibrium, the monopolist can maximize profit by setting sufficiently high per-interaction prices $p_H^* = p_L^*$ so that all agents match and exit in the first period and $\sigma_H^* = \sigma_L^*$.*

Proof. Appendix. □

One fact to note is that although the fixed prices are the same for both types of agents, the optimal per-interaction prices may or may not be the same. In particular, the only restriction on per-interaction prices is that they have to be higher than a certain threshold. After they reach the threshold, whether or not the per-interaction prices for studs and duds are the same is inconsequential.

Idiosyncratic Matching Shock ψ is Arbitrarily Distributed on $[0, m]$ We can prove few propositions about the general pricing dynamics for arbitrary distributions, but we now note that, generally, the platform prefers to allow low types in. The intuition for this is that, as we increase the inflow of low types, we can force them out at a proportionally faster rate by making them less selective with higher per-interaction prices, keeping the proportion of duds on the platform constant. Then the only threshold that will change is $\psi_L^*(L)$, since studs are unaffected. We can then show that the raw increase in the mass of low types dominates any decrease in profit per agent. The qualifications here are that the per-interaction price for high types cannot be too low relative to the price for low types, which could overturn Corollary 1, and that it low types don't accept all other low types, so that decreasing the threshold is actually possible.

Proposition 8. *Suppose $0 < i_L^* < i_L$, fix p_H and p_L, and suppose $\psi_H^*(L) \geq \psi_L^*(H)$ and $\psi_L^*(L) \in (0, m)$. The firm can strictly increase profits by increasing both i_L and p_L by $\epsilon > 0$.*

Proof. Appendix. □

Despite the small number of primitives in this model, analytical results are difficult to obtain. This is primarily due to two factors, the first being that the the effects of perturbations in the model depend on the exact specification of the distribution function of ψ, $F(.)$. The second reason is because of the fixed point nature of the equilibrium. In particular, even though agents take the proportion of high types on the platform, α, as given when optimizing, in equilibrium their thresholds will determine α. In particular, since prices affect thresholds, prices will then also affect α through both the thresholds of the agents and the outflow rates for both types. To explore the dynamics of the two-part tariff model, we conduct simulations for a range of parameters that illustrate the stylized facts of the model without getting into the complications of striving for analytical results. We present just a few simulations, as the dynamics are not particularly complex. Table 2 shows us optimal thresholds and prices under CRM when $\theta_H = 10$, $\theta_L = 1$, $I_L = 1$, and f is distributed U[0,1]. We see three regimes. For low values of r, it's optimal for agents to wait for high fit matches. Studs can't match to duds without accepting a broad range of fit values in stud matches–in this case, they must accept all studs to match to any duds–so any mixed strategy is extremely costly, and we have separating equilibria. Note that this does not show that studs rejecting all duds is the first best assignment. Rather, the two-part tariff instrument does not give the platform the flexibility to force stud-dud matches while retaining selective stud-stud matching; agent threshold strategies preclude this option. Given a separating equilibrium, intramatch externalties are eliminated, since every agent matches to their mirror image, and the prices we see exclusively reflect intermatch externalities, which we can classify as both congestion and thick market externalities. As one would expect, prices are negative for studs and positive for duds, increasing the value of α and giving studs faster matching via the thick market externality. This also imposes congestion of the low types, but their matches are less valuable.

For higher values of r, studs should be less selective and accept most or all stud-stud matches, so dropping the ψ_{HH} threshold is no longer costly, and the next regime sees some mixing with an interior value of ψ_{HL}. Intermatch externalities are still in play and militate towards positive prices for duds and negative prices for studs, but now intramatch externalities enter: studs are too picky when matching to duds, so there's an incentive for the platform to raise prices on studs. We can see that this effect dominates the intramatch externalities, yielding net positive prices for studs and duds. For very high values of r, the third regime features universal acceptance, as described above.

For LRM, Table 3 shows three regimes again. For low values of r, we have a separating equilibrium, and, as we show later in Proposition 12, a separating equilbrium under LRM is equivalent to two independent stratified platforms under LRM. Thus, there are no intramatch externalities across qualities, since the arrival rate of matches for studs is independent of the behavior of duds and vice versa. Instead, we see negative prices for both types reflecting the standard externality in LRM matching models–agents don't internalize that being pickier makes the market thicker, speeding up the arrival rate of draws. The second regime is much the same as under CRM, with intramatch externalities dominating and yielding positive prices. Finally, there is universal acceptance for sufficiently high r.

Implementation: A single two-part tariff schedule is easy to implement, only requiring experimentation in the use fee, or the length of contract (a month vs a year), a pricing strategy with similar dynamics that is common with online platforms. Price discriminating two-part tariffs may be illegal in some juristictions, and are difficult to implement optimally, as each type of agent's response to the price affects the others'. However, platforms like Tinder have implemented differential pricing for "duds" (in their case, older members).

5.1.3 Match-Dependent Pricing

We now consider a more complex pricing strategy: match-dependent pricing. By this we mean that, in addition to quality dependent fixed fees and per-interaction prices, the platform may charge a distinct price upon matching for each potential pairing. In practice, the platform need only set two such prices–one for matching to a high quality partner when you are high quality, and one for matching to a high quality partner when you are low quality[14]. With this flexibility, the platform can essentially transform the NTU environment into a TU environment by providing

[14]Generalizing to n quality levels, at most $(n-1)n$ such prices will be necessary.

the transfers that agents can't make themselves. Shimer and Smith (2001) show that, in a decentralized search-and-matching setting with heterogeneous agents, TU is not enough by itself to achieve first best. However, they show that the addition of per-interaction prices is sufficient to achieve first best in their model, and so it is with ours.

The intuition is as follows: the platform wants to implement the three optimal cutoffs ψ_{HH}, ψ_{HL}, and ψ_{LL}. The difference between an agent's cutoffs when matching to Studs and Duds is exactly the difference in match quality, since the agent's optimization is based on their continuation value, which is independent of the current draw. By adding an additional cost to matching to high types that gap will be increased (or decreased) by precisely the additional cost. Then, any $\psi_{HH} - \psi_{HL}$ and $\psi_{HL} - \psi_{LL}$ can be induced via match dependent pricing. Given those cutoff differences, appropriate per-interaction prices can be chosen to implement the desired lower cutoffs–ψ_{HL} and ψ_{LL}–giving us the complete cutoff scheme. Fixed fees soak up residual surplus in the monopoly case and ensure IR constraints are satisfied in the planner's case. Define ϕ_q as a price charged to agents of quality q upon matching to a high quality partner. Then the matching utility for agent i is $u(i,j) = \psi_{ij} + \theta_j - \phi_{q_i} I_{\theta_j = h}$.

Proposition 9. *There exist prices (σ, p, ϕ) maximizing TS.*

Proof. Appendix. □

Implementation: This sort of aggressive price discrimination may be difficult to implement in practice as it would involve not just differing prices for different users, but match dependent prices that would make it more costly to match to certain types of partners, discriminating against them along two dimensions. By construction, this price discrimination must be transparent–it is exactly the price differentials between different types of matches that induce optimal behavior. This makes it even more likely to irk users. This sort of pricing is also likely to be illegal in many jurisdictions. Finally, in a more general case with more than two quality grades, the number of prices to be determined increases quadratically, and, rather than setting cutoffs directly, the platform must simultaneously determine this menu of prices to induce the appropriate cutoffs. Thus, this instrument is technically challenging to implement as well.

5.2 Splitting Platforms

The strategies we've considered thus far have only utilized prices, but a platform may also use the structure of the market itself as an instrument to improve efficiency. An obvious and extremely simple approach is to split the platforms along the quality dimension, creating one market for Studs, and another for Duds. This can be either explicit, as in platforms like EliteSingles, or implicit, where a platform curates draws and simply excludes pairings between studs and duds. We find that, with CRM–that is, when platforms have constant returns to scale–splitting the platforms can always improve total surplus. The intuition is simple: agents match too much on the quality trait, and therefore reject good fit, low quality matches they should accept ("type I errors") and accept bad fit, high quality matches they should reject ("type II errors"). Segregating agents by quality avoids this temptation, and with CRM the rate of draws on each split platform is the same as on a mixed platform, making split platforms more efficient. This logic is complicated by the fact that the quality of one agent accrues to their partner, who, if they are of a different quality, may face a different average search time and therefore discount the payoff differently. However, our proof shows that this does not undermine the result. This proposition mirrors a similar result from Damiano and Li (2007), but for an entirely different reason. In Damiano and Li (2007), forcing assortation via platform partition is optimal because agents have supermodular utility, so matching high types with high types is more efficient than mixed matching–without partition, agents don't match closely enough on quality. In our case, however, partition prevents agents from matching too aggressively on quality to the detriment of fit. However, we show that this result depends strongly on the CRM assumption. If there are increasing returns to scale in platform size (LRM), it's always possible to generate more surplus on a single, mixed-quality platform because of the increased market thickness. Before we prove this proposition, it will be useful to note a variant of Jensen's Inequality for CDFs:

Lemma 5. *If*

$$F(\psi) = \alpha F(\psi_1) + (1 - \alpha) F(\psi_2),$$

then

$$\alpha \int_{\psi_1}^{m} x f(x)\, dx + (1 - \alpha) \int_{\psi_2}^{m} x f(x)\, dx \leq \int_{\psi}^{m} x f(x)\, dx$$

Proof. Appendix. □

Now define TS_m^* as the maximum total surplus for mixed platforms, and TS_s^* as the maximum for split platforms.

Proposition 10. *Given CRM, there exists cutoffs ψ such that $TS_s(\psi) \geq TS_m^*$.*

Proof. Appendix. □

We've shown that separate platforms can generate more total surplus with appropriate cutoffs, but we haven't shown that the decentralized outcome induces them. Clearly, any set of instruments that can achieve first best can achieve cutoffs satisfying this inequality, but the following result shows that even two-part tariffs can do so:

Proposition 11. *There exist prices (σ, p) inducing $TS_s^* > TS_m^*$.*

Proof. Appendix. □

Clearly, when CRM holds and there are no direct benefits to having a larger platform, a planner or monopolist should operate separate platforms for Studs and Duds. However, this does not hold under LRM. When the rate of draws increases in platform size, it will generally be preferable to aggregate users onto a single platform, assuming their matching behavior can be appropriately influenced. Specifically, with linear returns to matching, the rate of draws is proportional to the mass of agents on the platform–a mixed platform can have the same rate of Stud draws as a Studs-only platform while still having the same rate of Dud draws as a Duds-only platform. Therefore, if we assume search costs are low (s=0)[15], a mixed platform has no disadvantages relative to split platforms–even if Studs and Duds refuse to match to one another, they'll do just as well, and find matches just as fast, as if they were on separate platforms. The intuition of the proof comes from this insight: we can exactly replicate the total surplus of split platforms with a mixed platform where there are no cross-quality matches. From there, if we raise the Stud-Stud and Dud-Dud cutoffs, eliminating some lower fit matches, and add a proportional mass of Stud-Dud matches at the top of the fit distribution, we improve expected fit while holding expected time to match constant, strictly improving total surplus. This argument

[15] Gaining economies of scale from a single, large platform relies on giving users more draws and allowing them to be pickier. Search costs undermine that strategy, and the optimality of a single platform would depend on a comparison of the relative costs of search and the benefits of improved matching on fit. This can be overcome with censored search, as described in Section 5.3

is complicated by differing time discounting between studs and duds, but the proposition below provides a sufficient condition for the result to hold.

Proposition 12. *Given LRM and $s = 0$, $\exists$ cutoffs ψ such that $TS_m(\psi) > TS_s^*$ if $\theta_H + \psi_{hh}^* < \theta_L + m$ and $\theta_L + \psi_{ll}^* < \theta_H + m$*[16].

Proof. Appendix. □

Implementation: This is a comparatively easy strategy to implement, though with a continuum of quality the optimal number of platforms may be quite large (infinite with CRM, though the CRM assumption itself will inevitably break down at sufficient granularity.)

5.3 Censored Search

We now consider a second non-price instrument, censoring the draws an agent can observe. Curating agent choice sets is ubiquitous on online matching platforms – there are typically more partners available than can be presented at one time. Platforms will often limit the observable set of partners to those it considers a good match. At the very least, a platform will sort the set of potential matches, showing some earlier than others. Regardless, this can be framed as censoring, at least temporarily, some partners. Aside from lowering search costs and acting as a matching expert, our analysis thus far illuminates a third reason a platform may pursue this strategy: eliminating matching externalities. However, this instrument's reach is limited–it can't directly induce agents to accept matches they don't want. Censored search can eliminate "type II errors", but can't directly eliminate "type I errors". The cost of this limitation becomes obvious when we consider the externalities we've discussed thus far: generally, the platform wants Studs to be less picky when matching to Duds in order to accept good fit matches. An obvious approach using censored search would be to censor the lower fit Stud-Stud matches, forcing Studs to be less selective with Duds. However, direct inspection of the Stud's continuation value – which is also their expected utility from the platform – and corresponding indifference condition for a reservation value show that the cutoff for accepting a Dud is own continuation value plus a constant–to decrease a Stud's cutoff for Duds by x, the platform must lower their expected utility

[16] While this proposition depends on endogenous cutoffs, any parameterization of the model can be made to satisfy it by choosing a $p > m$ sufficiently large and an ϵ sufficiently small and replacing the fit distribution F with F^* such that $f^*(x) = (1-\epsilon)f(x) + \epsilon g_p(x)$, where g_p is the degenerate PDF with support only on p.

by x. In special cases where there is a mass point of Duds just below the Stud's cutoff, slightly lowering the expected utility for Studs to get Duds better matches can be optimal, but this is extremely sensitive to the fit distribution and the parameters of the model.

We'll now illustrate how, under the right circumstances, a platform can utilize censored search to improve total surplus. The benefit of lowering the Studs' cutoff is proportional to the mass of Duds who are now accepted by Studs that would have previously rejected them, so a Bernoulli fit distribution will illustrate a best case scenario for the potential use of the instrument. Formally, $\psi_{ij} \sim Bern(q)$. With censored search, we'll assume the platform can observe both type and fit between any pair. This is not as strong an assumption as it may seem, as, while lying about quality can potentially generate better matches, lying about fit parameters typically does not. We'll first establish a few properties of the decentralized equilibrium with binary fit, then find parameter ranges where censoring bad fit Stud-Stud matches is surplus improving. Note that every draw that is censored generates no search costs, and incurs no per-interaction price – these are potential draws that are never actually realized. For concision, we'll denote a match by $\theta_i\theta_j\psi_{ij}$. To avoid the multiplicity of cases and equilbria that can occur in this setting, we'll focus on a case where $\theta_l = 0$, and $i_L = 1$. We'll assume $s = 0$ to simplify the interpretation of our results – if s is positive, the platform can benefit from the more obvious advantage of censored search: eliminating search costs by censoring draws that will be rejected anyway. With $s = 0$, that channel is shut down and the platform can only utilize censored search to change users' matching outcomes. We'll also assume that indifferent matches are always accepted.

Lemma 6. *In the decentralized equilibrium, $HH1$ and $LL1$ must be accepted, while $HL0$ and $LL0$ are always rejected.*

Proof. Appendix. □

Proposition 13. *There exist parameters such that censoring HH0 draws improves total surplus, but for sufficiently high $\frac{h}{r}$, censoring $HH0$ draws cannot improve total surplus.*

Proof. Appendix. □

5.3.1 Adding per-interaction costs

It seems that, by itself, censored search is of ambiguous value. There are cases where it can improve total surplus, but they are very sensitive to the parameters of the model. However, with the addition of per-interaction prices, censored search is a much more powerful tool In particular, positive per-interaction costs act as additional search costs, forcing agents to be less selective. This eliminates "type I errors" at the cost of generating more "type II errors". Since censored search can eliminate the latter but not the former, this gives censored search more traction. With high enough per-interaction costs, all "type I errors" are eliminated, and the platform can achieve first best.

Proposition 14. *There exist prices* $(\sigma, \mathbf{p})$ *and censoring cutoffs* ψ_c *maximizing total surplus.*

Proof. Appendix. □

Implementation: Unlike the pure pricing instruments, censored search only requires that the platform be able to identify "bad" matches–they don't need to devise a menu of prices to indirectly induce ideal matching. Even when adding a per-interaction cost, the platform need only ensure that it is sufficiently large, and does not need to price discriminate. This suggests that censored search can both be extremely powerful and easily implementable. Even if extremely high per-interaction prices are infeasible in practice, any increase in per-interaction costs gives censored search more traction, illustrating the complementarity of these two instruments.

6 Conclusion

This paper presents a multidimensional matching model with search where economic agents are differentiated both in terms of *quality* and *fit*. In presenting a more complex modeling of preferences, we highlight a previously unstudied source externalities – the tradeoff between quality and fit. We study how a strategic platform can leverage a variety of instruments to improve surplus on the platform, and find the combination of censored search and simple two-part tariffs particularly effective in our setting. Our model is broad, covering constant and increasing returns in the matching technology as well as both of the common specifications for search frictions – search costs and time discounting. We also include extensions to frictionless matching. However, our model is extremely simple, and thus suggests a variety of directions for further research. One promising direction is competition – our model assumes a monopoly, but many matching platforms operate in a competitive setting. Modular utility simplifies our analysis greatly, but supermodularity may be more realistic for many applications, and may change the welfare analysis for some instruments. Finally, we assume a setting where the platform can utilize fixed fees to perfectly price discriminate. This, along with modular utility, ensures that incentive compatability constraints will always be satisfied. Relaxing these assumptions would allow a rich analysis of the constraints incentive compatibility may place on a strategic matching platform in this environment.

Appendix

Lemma 2:

Proof. Clearly, $(\theta_H + m) + i_L(\theta_L + m)$ is an upper bound for the TSS, so the function is bounded above. Outflow is continuous in ψ and TSS is continuous in outflow, so TSS is continuous in ψ. Finally, the domain of ψ is compact. Thus, TSS achieves a maximum in Ψ. □

Proposition 3:

Proof. Note that, per Equation 4, $\psi_H(\theta)$ is increasing in C_H and $\psi_L(\theta)$ is increasing in C_L, then

1. Without loss of generality, consider the high type's cutoffs. By Equation 5, $\frac{\partial C_H}{\partial s} = -\frac{1}{r/\lambda + o_H} < 0$

2. Without loss of generality, consider the high type's cutoffs. By Equation 5, $\frac{\partial C_H}{\partial r} = -\frac{C_H}{(r + o_H\lambda)} < 0$

3. By Equation 5, $\frac{\partial C_H}{\partial \alpha} \leq 0$ iff $\int_{\psi^*_H(H)}^{a} (\theta_H + \psi)\, f(\psi)\, d\psi \geq \int_{\psi^*_H(L)}^{a} (\theta_L + \psi)\, f(\psi)\, d\psi$. This must hold since $\psi^*_H(L) \geq \psi^*_H(H)$.

4. Corollary to the previous result.

□

Proposition 4:

Proof. Total surplus is then

$$\sum_{i=1}^{n} u_i = \sum_{i=1}^{n} (\sum_{j=1}^{k} (\theta^m_{i,j} + \theta^w_{\mu(i),j}) + f(\psi^m_i, \psi^w_{\mu(i)})) = \sum_{i=1}^{n} (\sum_{j=1}^{k} (\theta^m_{i,j} + \theta^w_{i,j}) + f(\psi^m_i, \psi^w_i))$$

for any μ. For assignments μ and η, let TSS_μ and TSS_η and denote the overall total social surplus generated from the assignments (i.e., the total social surplus from both the vertical and heterogenous dimensions; for example, $\text{TSS}_\mu = \text{TSS}_{\mu,\theta} + \text{TSS}_{\mu,\psi}$). Then,

$$\text{TSS}_\mu - \text{TSS}_\eta = \text{TSS}_{\mu,\theta} - \text{TSS}_{\eta,\theta} + \text{TSS}_{\mu,\psi} - \text{TSS}_{\eta,\psi} = \text{TSS}_{\mu,\psi} - \text{TSS}_{\eta,\psi}$$

and, since transferable utility stable matchings maximize TSS by Shapley and Shubik (1971), the transferable stable matching μ^* must satisfy

$$\mu^* = \underset{\mu}{\operatorname{argmax}} \, \mathrm{TSS}_{\mu,\theta} + \underset{\mu}{\operatorname{argmax}} \, \mathrm{TSS}_{\mu,\psi} = \underset{\mu}{\operatorname{argmax}} \, \mathrm{TSS}_{\mu,\psi}$$

□

Lemma 3:

Proof. Let $\psi'_{HL} = max\{\psi_{HL}, \psi_{LH}\}$. A Stud-Stud or Dud-Dud match is accepted under ψ' if is accepted under ψ, as the cutoffs are identical. A match between a Stud and a Dud is accepted under ψ iff $\psi_{ij} \geq max\{\psi_{HL}, \psi_{LH}\}$. Then, given that a match is accepted under ψ' iff $\psi_{ij} \geq \psi'_{HL} = max\{\psi_{HL}, \psi_{LH}\}$, the result is proved. □

Lemma 4:

Proof. Suppose a draw will be rejected by one of the agents. Then the platform can improve utility by censoring the draw. Then every uncensored draw will be accepted, and search cost per agent will be s. s is constant in ψ, so ψ^* is invariant to s.

□

Proposition 5:

Proof. First, consider agents' strategy profiles. There are $(2^2)^2 = 16$ relevant profiles, given that high and low types can choose acceptance or rejection for the two types they can meet. However, in equiliibrium all agents must always accept high types as the continuation value must be strictly less than the high type matching payoff. Further, by Corollary 1 low types must accept low types if high types accept low types, and if high types reject low types, the payoff for matching to a low type must be strictly higher than the low type continuation value. Thus the only two relevant strategy profiles are all agents accepting every match (pooling) and high types rejecting low types and accepting high types while low types accept all matches (separating). Applying these profiles to the definition of outflow and Equation 11, we find that $o_L = \frac{\sqrt{i_L}}{1+\sqrt{i_L}}, o_H = \frac{1}{1+\sqrt{i_L}}$ and $\alpha = \frac{1}{1+\sqrt{i_L}}$ for the separating case and $o_L = o_H = 1$ and $\alpha = \frac{1}{1+i_L}$ for the pooling case. Note that

$$TSS_{CRM} = \frac{o_H \theta_H - s}{r + o_H} + i_L \frac{o_L \theta_L - s}{r + o_L}$$

and

$$TSS_{LRM} = \frac{o_H\theta_H - s}{\frac{r}{\sqrt{\frac{1}{o_H}+\frac{i_L}{o_L}}} + o_H} + i_L\frac{o_L\theta_L - s}{\frac{r}{\sqrt{\frac{1}{o_H}+\frac{i_L}{o_L}}} + o_L}$$

For CRM, holding inflow constant, direct inspection shows TSS is increasing in o_L and o_H, so pooling generates higher TSS. For LRM, holding inflow constant, we'll show this by proving that the expected time to exit, defined as $e_H = \frac{1}{o_H\lambda}$ and $e_L = \frac{1}{o_L\lambda}$, is lower for the pooling case. We can rewrite

$$TSS = \frac{\theta_H}{1+r/e_H} + i_L\frac{\theta_L}{1+r/e_L} - s(\frac{1}{o_H(1+r/e_H)} + \frac{1}{o_L(1+r/e_L)})$$

In the pooling case, $e_H = e_L = \frac{1}{\sqrt{1+i_L}}$, while in the separating case $e_H = 1$ and $e_L = \frac{1}{\sqrt{i_L}}$, both greater than the pooling case. Since o_H and o_L are also greater in the pooling case, the TSS must be greater in the pooling case. Prices exist that induce such an acceptance strategy: $p_H = p_L > \theta_H - \theta_L$. Direct inspection shows that TSS is increasing in inflows. Then the platform's optimal strategy is to set prices to induce outflow rates of 1 and to allow all inflow agents onto the platform.

□

Proposition 6:

Proof. Define the universal acceptance strategy S^* and fix an agent strategy profile S' with $\psi^* > 0$—that is, at least one cutoff binds. Define the set of pairings that generate acceptance as A and the complement R. Holding inflows constant, consider the surplus generated by agents who enter at time t under both strategy profiles. The mass of agents who receive first draws in A and generate the same surplus in each case. We need only concern ourselves with those who receive first draws in R. The expected contribution to surplus generated by the agents in R accepting their first draw corresponds to their own quality plus their fit component, bounded below by $(\theta_H(1-o_H) + \theta_L i_L(1-o_L))/(1+r)$. If the agents continue to search, the best they can hope for is perfect fit matches, and their contribution to expected surplus is equivalent to an agent of the same quality who joined the platform at the time of their first draw–that is, discounted by $1/(1+r)$ in expectation. Then there is a probability $\frac{\alpha F(\psi_{HH})}{o_H+r}$ a stud will match to a stud and contribute $\theta_H + m$, discounted by a stud's average discount at time of match $\frac{o_H}{o_H+r}$ and a probability $\frac{(1-\alpha)F(\psi_{HL})}{o_H+r}$ a stud will match to a dud and contribute $\theta_H + m$ discounted by a stud's average discount at time of match $\frac{o_L}{o_L+r}$. Given a symmetric situation for the duds, expected contribution to total surplus is

$$
\begin{aligned}
= \frac{1}{1+r}((1-o_H)(\theta_H+m)(\frac{o_H}{o_H+r}\frac{\alpha F(\psi_{HH})}{o_H}+\frac{o_L}{o_L+r}\frac{(1-\alpha)F(\psi_{HL})}{o_H}) \\
+(\theta_L+m)(\frac{o_H}{o_H+r}i_L\frac{\alpha F(\psi_{HL})}{o_L}+\frac{o_L}{o_L+r}i_L\frac{(1-\alpha)F(\psi_{LL})}{o_L})) \quad (16)
\end{aligned}
$$

Exploiting the fact that the probabilities of matching to a stud or dud sum to 1, we have

$$= \frac{1}{1+r}((1-o_H)(\theta_H+m)(\frac{o_H}{o_H+r}m_{HH}+\frac{o_L}{o_L+r}(1-m_{HH}))+(\theta_L+m)i_L(\frac{o_H}{o_H+r}m_{HL}+\frac{o_L}{o_L+r}(1-m_{HL})))$$

Where m_{qp} are the probabilities. $o_H, o_L \leq 1$, so this is weakly less than

$$\frac{1}{1+r}((1-o_H)(\theta_H+m)(\frac{1}{1+r}+(\theta_L+m)i_L\frac{1}{1+r})$$

the TSS for immediately leaving on the first expected draw with a perfect fit match. It is immediate from the fact that all agents find a partner that total contributions to surplus equal TSS, so unconditional acceptance is TSS maximizing if $m < r\theta_L$ and $m < r\theta_H$. Only the former binds. Finally, by direct inspection, $TSS_{S*} = \frac{\theta_H+\int_0^m f(\psi)d\psi+(\theta_L+\int_0^m f(\psi)d\psi)i_L}{1+r}$ is maximized when inflows are maximized.

□

Proposition 7:

Proof. We've established that universal acceptance maximizes platform profit, so per-interaction prices must be sufficient to induce universal acceptance. Studs and duds both receive an expected match surplus of

$$C = \frac{1/(1+i_L)\int_0^m(x+\theta_H)f(x)dx + i_L/(1+i_L)\int_0^m(x+\theta_L)f(x)dx - s}{r+o}$$

. A per-interaction fee will rationalize universal acceptance if

$$\theta_L > \frac{1/(1+i_L)\int_{\psi_H}^m(x+\theta_H)f(x)dx + i_L/(1+i_L)\int_{\psi_L}^m(x+\theta_L)f(x)dx - s - p}{r+o}$$

for all thresholds, which holds for positive p by Proposition 6. Then $\sigma = C - \frac{p}{r+o}$ extracts consumer surplus for studs and duds. □

Proposition 8:

Proof. Fix $\boldsymbol{\psi}^*$ and i_L^*, suppose $C_H > C_L$ and $\psi_{LL} \in (0, m)$. Then TSS can be increased by increasing i_L and decreasing ψ_{LL}

First, note that $\alpha = 1/(1 + o_H i_L/o_L)$. To make the analysis tractible, we'll hold α,ψ_{HH}, and ψ_{HL} constant as we increase i_L, so we must have $i_L = co_L$ for some c. Equivalently, $i_L = c(\alpha \int_{\psi^*_{HL}}^m f(x)dx + ((1-\alpha)\int_{\psi'_{LL}}^m f(x)dx))$. Thus, given that $f(x) > \delta > 0$, ψ_{LL} is a continuous decreasing function of i_L within an ϵ-ball of i_L^*. Define $A \equiv \frac{(i_L)}{r+o_L}$ and $B \equiv \alpha \int_{\psi_{HL}}^m (x+\theta_H)f(x)dx + ((1-\alpha)\int_{\psi_{LL}}^m (x+\theta_L)f(x)dx$. The C_H component of TSS is invariant to changes in i_L and ψ_{LL}, since neither enters that expression, so we'll consider only

$i_L^* C_L^* = A^* B^*$. Now consider $i'_L = i_L^* + \epsilon$,ψ'_{LL} and o'_L satifying the above. This yields

$i'_L C'_L = A'B'$,

The lefthand term $A = \frac{(i_L)}{r+(i_L)c}$ is increasing in i_L, so showing that B is increasing in i_L in will be sufficient to show that $i_L C_L$ is and thus that TSS is. In fact, B' can be rewritten $\alpha \int_{\psi^*_{HL}}^m (x+\theta_H)f(x)dx + ((1-\alpha)(\int_{\psi^*_{LL}}^m (x+\theta_L)f(x)dx + \int_{\psi'_{LL}}^{\psi^*_{LL}} (x+\theta_L)f(x)dx)) = B^* + (1-\alpha)\int_{\psi'_{LL}}^{\psi^*_{LL}} (x+\theta_L)f(x)dx) > B^*$.

□

Proposition 9:

Proof. By Lemmas 2 and 3, there exists $\psi^* = (\psi^*_{HH}, \psi^*_{HL}, \psi^*_{LH}, \psi^*_{LL})$ such that $\psi^*_{LH} = \psi^*_{HL}$ and ψ^* maximizes total surplus. Let $d_h \equiv \psi^*_{HH} - \psi^*_{HL}$ and $d_l \equiv \psi^*_{HL} - \psi^*_{LL}$. Let $d \equiv h - l$. Then set high type match dependent price $\phi_h = d - d_h$ and low type match dependent price $\phi_l = d - d_l$. Then $\psi^*_{HH} - \psi^*_{HL} = \psi_{HH} - \psi_{HL}$ and $\psi^*_{HL} - \psi^*_{LL} = \psi_{HL} - \psi_{LL}$. Now let p_h be such that $\psi^*_{HL} = EU_H[p_h]$ and let p_l be such that $\psi^*_{LL} = EU_L[p_l]$. Choose $\sigma_i = EU_i(\psi^*, p, \phi)$. Then $EU_i(\psi^*, p, \phi, \sigma) = 0$.

□

Lemma 5:

Proof. Suppose $F(\psi) = \alpha F(\psi_1) + (1-\alpha)F(\psi_2)$

We want to show

$$\alpha \int_{\psi_1}^m x f(x)\, dx + (1-\alpha)\int_{\psi_2}^m x f(x)\, dx - \int_{\psi}^m x f(x)\, dx \leq 0$$

Applying integration by substitution, we have

$$\alpha \int_{F(\psi_1)}^{m} F^{-1}(y)\,dy + (1-\alpha)\int_{F(\psi_2)}^{m} F^{-1}(y)\,dy - \int_{F(\psi)}^{m} F^{-1}(y)\,dy$$

without loss of generality, assume $\psi_1 \geq \psi_2$. Cancelling overlapping regions of integration, we have

$$-\alpha \int_{F(\psi)}^{F(\psi_1)} F^{-1}(y)\,dy + (1-\alpha)\int_{F(\psi_2)}^{F(\psi)} F^{-1}(y)\,dy$$

F^{-1} is increasing as a well defined CDF, so

$$-\alpha \int_{F(\psi)}^{F(\psi_1)} F^{-1}(y)\,dy + (1-\alpha)\int_{F(\psi_2)}^{F(\psi)} F^{-1}(y)\,dy \leq -\alpha \int_{F(\psi)}^{F(\psi_1)} F^{-1}(\psi)\,dy + (1-\alpha)\int_{F(\psi_2)}^{F(\psi)} F^{-1}(\psi)\,dy$$

Then it sufficies to show the following is less than zero:

$$-\alpha \int_{F(\psi)}^{F(\psi_1)} dy + (1-\alpha)\int_{F(\psi_2)}^{F(\psi)} dy$$

$$= -\alpha(F(\psi_1) - F(\psi)) + (1-\alpha)(F(\psi) - F(\psi_2))$$

By assumption,

$$= -\alpha(F(\psi_1) - (\alpha F(\psi_1) + (1-\alpha)F(\psi_2))) + (1-\alpha)((\alpha F(\psi_1) + (1-\alpha)F(\psi_2)) - F(\psi_2))$$

$$= -\alpha((1-\alpha)(F(\psi_1) - F(\psi_2))) + \alpha(1-\alpha)((F(\psi_1) - F(\psi_2))) \leq 0.$$

□

Proposition 10:

Proof. Define ψ^* as a set of cutoffs inducing TS_m^*. We'll proceed by cases, starting with case 1: assume $o_L \leq o_H$. Define $\psi_H \equiv F^{-1}(\alpha F(\psi^*_{HH}) + (1-\alpha)(F(\psi^*_{HL})))$ and $\psi_L \equiv F^{-1}(\alpha F(\psi^*_{HL}) + (1-\alpha)F(\psi^*_{LL}))$. For feasibility, the quality and search cost components of TSS_m is of the form

$$c\frac{o_H\theta_H - s}{o_H + r} + (1-c)\frac{o_L\theta_H - s}{o_L + r} + (d\frac{o_H\theta_L - s}{o_H + r} + (1-d)\frac{o_L\theta_L - s}{o_L + r})i_L$$

for $c, d \in [0,1]$ and $(1-c) = di_L$, the corresponding component of TSS_s is

$$i_L \frac{o_L\theta_L - s}{o_L + r} + \frac{o_H\theta_H\text{-s}}{o_H + r}$$

Subtracting the former from the latter, and substituting out for c, we have

$$di_L(\theta_H - \theta_L)(\frac{o_H}{o_H + r}\text{-}\frac{o_L}{o_L + r}) \geq 0$$

Thus, the quality and search component of TSS is greater for the split platform.

The fit component is

$$\frac{i_L\left(\alpha\int_{\psi_{HL}}^m xf(x)\,dx - (\alpha-1)\int_{\psi_{LL}}^m xf(x)\,dx\right)}{o_L + r} + \frac{\alpha\int_{\psi_{HH}}^m xf(x)\,dx - (\alpha-1)\int_{\psi_{HL}}^m xf(x)\,dx}{o_H + r}$$

for mixed vs

$$\frac{i_L\int_{\psi_L}^m f(x)\,(x)\;dx}{o_L + r} + \frac{\left(\int_{\psi_H}^m f(x)\,(x)\;dx\right)}{o_H + r}$$

for split, which is larger by Lemma 5, completing the proof.

Case 2: $o_L > o_H$

Suppose both split platforms share a cutoff ψ such that

$$\frac{\int_\psi^m f(x)dx}{r+\int_\psi^m f(x)dx}(1+i_L) =$$
$$\frac{\alpha\int_{\psi_{HH}}^m f(x)dx + (1-\alpha)\int_{\psi_{HL}}^m f(x)dx}{r + (\alpha\int_{\psi_{HH}}^m f(x)dx + (1-\alpha)\int_{\psi_{HL}}^m f(x)dx)} + i_L\frac{\alpha\int_{\psi_{HH}}^m f(x)dx + (1-\alpha)\int_{\psi_{HL}}^m f(x)dx}{r + (\alpha\int_{\psi_{HL}}^m f(x)dx + (1-\alpha)\int_{\psi_{LL}}^m f(x)dx)} \quad (17)$$

Note that the quality and search component of TSS for the split platform is

$$UQ_s = \frac{\int_\psi^m f(x)dx}{r+\int_\psi^m f(x)dx}(\theta_H + \theta_L i_L) - \frac{s}{r+\int_\psi^m f(x)dx}(1+i_L)$$

. By construction,

$$\frac{s}{r+\int_\psi^m f(x)dx}(1+i_L) = \frac{s}{\alpha\int_{\psi_{HH}}^m f(x)dx + (1-\alpha)\int_{\psi_{HL}}^m f(x)dx} + \frac{si_L}{\alpha\int_{\psi_{HH}}^m f(x)dx + (1-\alpha)\int_{\psi_{HL}}^m f(x)dx}$$

, so the search costs are identical between split and mixed platforms.

Defining $d_L = \frac{o_L}{r+o_L}$ and $d_H = \frac{o_H}{r+o_H}$, we can rewrite the quality component of TSS_s as

$$\theta_H(d_H + i_L d_L)/(1+i_L) + i_L\theta_L(d_H + i_L d_L)/(1+i_L)$$

. Thus, the time discounting is a weighted average of d_H and d_L. We'll show that, under mixed platforms, high types receive a higher weighting for d_H and low types receive a lower weighting for d_H, so, since $d_L > d_H$, the expected discounting is higher for mixed platforms.

We can rewrite the quality component of TSS_m as

$$\theta_H(d_H(1-X)+d_LX)+\theta_L i_L(d_HX/i_L+d_L(1-X/i_L))$$

, where X is the proportion of high types that match to low types. Note that, if $X = i_L/(1+i_L)$, the quality component of TSS is equal between mixed and split platforms.

Now we'll show that $X < i_L/(1+i_L)$. Note that

$$\alpha = o_L/(o_H * i_L + o_L) > o_H/(o_H * i_L + o_H) = 1/(1+i_L)$$

and $X = \frac{(1-\alpha)\int_{\psi_{HL}}^m f(x)dx}{\alpha\int_{\psi_{HH}}^m f(x)dx+(1-\alpha)\int_{\psi_{HL}}^m f(x)dx}$ is decreasing in α, so

$$\begin{aligned} X &= \frac{(1-\alpha)\int_{\psi_{HL}}^m f(x)dx}{\alpha\int_{\psi_{HH}}^m f(x)dx+(1-\alpha)\int_{\psi_{HL}}^m f(x)dx} \\ &< \frac{i_L/(1+i_L)\int_{\psi_{HL}}^m f(x)dx}{1/(1+i_L)\int_{\psi_{HH}}^m f(x)dx+i_L/(1+i_L)\int_{\psi_{HL}}^m f(x)dx} \\ &\leq \frac{i_L/(1+i_L)\int_{\psi_{HL}}^m f(x)dx}{1/(1+i_L)\int_{\psi_{HL}}^m f(x)dx+i_L/(1+i_L)\int_{\psi_{HL}}^m f(x)dx} = i_L/(1+i_L) \end{aligned}$$

Then

$$\frac{\partial}{\partial x}\theta_H(d_H(1-X)+d_LX)+\theta_L i_L(d_HX/i_L+d_L(1-X/i_L)) = (\theta_H-\theta_L)(d_L-d_H) > 0$$

, so the quality component of TSS is greater for the split platform.

Now we'll treat the fit component to TSS.

$UF_s - UF_m$=
$\frac{\int_\psi^m xf(x)dx-(\alpha\int_{\psi_{HH}}^m xf(x)dx+(1-\alpha)\int_{\psi_{HL}}^m xf(x)dx)}{r+(\alpha\int_{\psi_{HH}}^m f(x)dx+(1-\alpha)\int_{\psi_{HL}}^m f(x)dx)}+i_L\frac{\int_\psi^m xf(x)dx-(\alpha\int_{\psi_{HL}}^m xf(x)dx+(1-\alpha)\int_{\psi_{LL}}^m xf(x)dx)}{r+(\alpha\int_{\psi_{HL}}^m f(x)dx+(1-\alpha)\int_{\psi_{LL}}^m f(x)dx)}$

We know $\psi_{HL} \geq \psi_{HH}, \psi_{LL}$. We can also show that $\psi \in [Min(\psi_{HH}, \psi_{HL}, \psi_{LL}), Max(\psi_{HH}, \psi_{HL}, \psi_{LL})]$. Thus, $\psi \leq \psi_{HL}$ and $\psi \geq \psi_{HH}$ or $\psi \geq \psi_{LL}$. First, suppose $\psi \geq \psi_{HH}$ and $\psi \geq \psi_{LL}$. Cancelling overlapping regions of integration, we have
$\frac{(1-\alpha)\int_\psi^{\psi_{HL}} xf(x)dx-(\alpha\int_{\psi_{HH}}^\psi xf(x)dx)}{r+(\alpha\int_{\psi_{HH}}^m f(x)dx+(1-\alpha)\int_{\psi_{HL}}^m f(x)dx)}+i_L\frac{\alpha\int_\psi^{\psi_{HL}} xf(x)dx-((1-\alpha)\int_{\psi_{LL}}^\psi xf(x)dx)}{r+(\alpha\int_{\psi_{HL}}^m f(x)dx+(1-\alpha)\int_{\psi_{LL}}^m f(x)dx)}$

which must be less than

$\frac{(1-\alpha)\int_{\psi}^{\psi_{HL}}\psi f(x)dx-(\alpha\int_{\psi_{HH}}^{\psi}\psi f(x)dx)}{r+(\alpha\int_{\psi_{HH}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{HL}}^{m}f(x)dx)}+i_L\frac{\alpha\int_{\psi}^{\psi_{HL}}\psi f(x)dx-((1-\alpha)\int_{\psi_{LL}}^{\psi}\psi f(x)dx)}{r+(\alpha\int_{\psi_{HL}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{LL}}^{m}f(x)dx)}$

collecting ψ terms we have

$$\psi(\frac{(1-\alpha)\int_{\psi}^{\psi_{HL}}f(x)dx-(\alpha\int_{\psi_{HH}}^{\psi}f(x)dx)}{r+(\alpha\int_{\psi_{HH}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{HL}}^{m}f(x)dx)}+i_L\frac{\alpha\int_{\psi}^{\psi_{HL}}f(x)dx-((1-\alpha)\int_{\psi_{LL}}^{\psi}f(x)dx)}{r+(\alpha\int_{\psi_{HL}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{LL}}^{m}f(x)dx)}) \tag{18}$$

We have the following identities:
$\frac{\int_{\psi}^{m}f(x)dx}{r+\int_{\psi}^{m}f(x)dx}(1+i_L)=\frac{\alpha\int_{\psi_{HH}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{HL}}^{m}f(x)dx}{r+(\alpha\int_{\psi_{HH}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{HL}}^{m}f(x)dx)}+i_L\frac{\alpha\int_{\psi_{HH}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{HL}}^{m}f(x)dx}{r+(\alpha\int_{\psi_{HL}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{LL}}^{m}f(x)dx)}$ and
$\int_{\psi}^{m}f(x)dx=\frac{o_Li_L(o_H+r)+o_H(o_L+r)}{i_L(o_H+r)+o_L+r}$, which yield
$\int_{\psi}^{\psi_{HL}}f(x)dx=\frac{o_Li_L(o_H+r)+o_H(o_L+r)}{i_L(o_H+r)+o_L+r}$-$\int_{\psi_{HL}}^{m}f(x)dx$ and
$\int_{\psi_{LL}}^{\psi}f(x)dx=\int_{\psi_{LL}}^{m}f(x)dx-\frac{o_Li_L(o_H+r)+o_H(o_L+r)}{i_L(o_H+r)+o_L+r}$

Then expression 18 can be rewritten as

$$\psi(\frac{(1-\alpha)(X-\int_{\psi_{HL}}^{m}f(x)dx)-(\alpha(\int_{\psi_{HH}}^{m}f(x)dx-X))}{r+(\alpha\int_{\psi_{HH}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{HL}}^{m}f(x)dx)}+\frac{\alpha((X-\int_{\psi_{HL}}^{m}f(x)dx))-((1-\alpha)(\int_{\psi_{LL}}^{m}f(x)dx-X)}{r+(\alpha\int_{\psi_{HL}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{LL}}^{m}f(x)dx)}$$

, where $X=\frac{o_Li_L(o_H+r)+o_H(o_L+r)}{i_L(o_H+r)+o_L+r}$

, which simplifies to zero.

Now, without loss of generality, assume $\psi_{HH}>\psi$, $\psi_{HH}\leq\psi$. Then we can rewrite UF_s-UF_m as
$\frac{(1-\alpha)\int_{\psi_{HH}}^{\psi_{HL}}xf(x)dx+\int_{\psi}^{\psi_{HH}}xf(x)dx}{r+(\alpha\int_{\psi_{HH}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{HL}}^{m}f(x)dx)}+\frac{\alpha\int_{\psi}^{\psi_{HL}}xf(x)dx-((1-\alpha)\int_{\psi_{LL}}^{\psi}xf(x)dx)}{r+(\alpha\int_{\psi_{HL}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{LL}}^{m}f(x)dx)}i_L$

Which must be less than
$\frac{(1-\alpha)\int_{\psi_{HH}}^{\psi_{HL}}\psi f(x)dx+\int_{\psi}^{\psi_{HH}}\psi f(x)dx}{r+(\alpha\int_{\psi_{HH}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{HL}}^{m}f(x)dx)}+\frac{\alpha\int_{\psi}^{\psi_{HL}}\psi f(x)dx-((1-\alpha)\int_{\psi_{LL}}^{\psi}\psi f(x)dx)}{r+(\alpha\int_{\psi_{HL}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{LL}}^{m}f(x)dx)}i_L$

Which can be rewritten as
$\psi(\frac{(1-\alpha)\int_{\psi_{HH}}^{\psi_{HL}}f(x)dx+\int_{\psi}^{\psi_{HH}}f(x)dx}{r+(\alpha\int_{\psi_{HH}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{HL}}^{m}f(x)dx)}+\frac{\alpha\int_{\psi}^{\psi_{HL}}f(x)dx-((1-\alpha)\int_{\psi_{LL}}^{\psi}f(x)dx)}{r+(\alpha\int_{\psi_{HL}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{LL}}^{m}f(x)dx)}i_L)$
$=\psi(\frac{(1-\alpha)\int_{\psi}^{\psi_{HL}}f(x)dx+\alpha\int_{\psi}^{\psi_{HH}}f(x)dx}{r+(\alpha\int_{\psi_{HH}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{HL}}^{m}f(x)dx)}+\frac{\alpha\int_{\psi}^{\psi_{HL}}f(x)dx-((1-\alpha)\int_{\psi_{LL}}^{\psi}f(x)dx)}{r+(\alpha\int_{\psi_{HH}}^{m}f(x)dx+(1-\alpha)\int_{\psi_{HL}}^{m}f(x)dx)}i_L)$, which is expression 18 and equal to zero. □

Proposition 11:

Proof. By Proposition 10, there exists $TS_s>TS_m^*$, so $TS_s^*>TS_m^*$. Given a (ψ_H^*,ψ_L^*) inducing TS_s^*, let p_h be such that $\psi_H^*=EU_H[p_h]$ and let p_l be such that $\psi_L^*=EU_L[p_l]$. Choose $\sigma_i=EU_i(\psi^*,p,\phi)$. Then (σ,p) induces TS_s^*. □

Proposition 12:

Proof. First, we'll show that, given $s = 0$, a mixed platform with no stud-dud matches generates payoffs equal to those of a split platform. That is,

$$\frac{\alpha \int_{\psi_{HH}}^{m} (x+\theta_h) f(x) dx}{r/\sqrt{i_L/o_L + 1/o_H} + o_H} + i_L \frac{(1-\alpha) \int_{\psi_{LL}}^{m} (x+\theta_l) f(x) dx)}{r/\sqrt{i_L/o_L + 1/o_H} + o_L} = \frac{\int_{\psi_{HH}}^{m} (x+\theta_h) f(x) dx}{r/\sqrt{i_L/o_{hs}} + o_{hs}} + i_L \frac{\int_{\psi_{LL}}^{m} (x+\theta_l) f(x) dx)}{r/\sqrt{i_L/o_{ls}} + o_{ls}} \tag{19}$$

where $o_{ls} = \int_{\psi_{LL}}^{m} f(x)dx = o_L/(1-\alpha)$ and $o_{hs} = \int_{\psi_{HH}}^{m} f(x)dx = o_H/\alpha$. Then, substituting the outflows, the split platform payoff is

$$\frac{\int_{\psi_{HH}}^{m} (x+\theta_h) f(x) dx}{r/\sqrt{i_L/o_{hs}} + o_{hs}} + i_L \frac{\int_{\psi_{LL}}^{m} (x+\theta_l) f(x) dx)}{r/\sqrt{i_L/o_{ls}} + o_{ls}} = \frac{\alpha \int_{\psi_{HH}}^{m} (x+\theta_h) f(x) dx}{r/\sqrt{i_L/(\alpha o_H)} + o_H} + i_L \frac{(1-\alpha) \int_{\psi_{LL}}^{m} (x+\theta_l) f(x) dx)}{r/\sqrt{i_L/((1-\alpha) o_L)} + o_L}$$

Direct inspection shows that this is equal to the right-hand side of Equation 19 if

$$i_L/((1-\alpha)o_L) = i_L/(\alpha o_H) = i_L/o_L + 1/o_H$$

In fact this holds because $\alpha = \frac{o_L}{o_L + i_L o_H}$.

This result in hand, we'll now use the TSS expression for a mixed platform with no stud-dud matches as TSS_s^* and compare it to a mixed platform with a nonzero amount of mixing, where $\psi_{HL} = m - \epsilon$ for $\epsilon > 0$, and ψ_{HH} and ψ_{LL} are such that outflows are held constant:

$$\alpha(1 - F(\psi_{HH})) + (1-\alpha)(1 - F(\psi_{HL})) = \alpha(1 - F(\psi_{HH}^*))$$

$$\alpha(1 - F(\psi_{HL})) + (1-\alpha)(1 - F(\psi_{LL})) = \alpha(1 - F(\psi_{LL}^*))$$

Such binding cutoffs exist, as F is continuous, $\alpha \in (0,1)$, and $\psi_{LL}^*, \psi_{HH}^* < m$. $TSS_m - TSS_s^*$ is then

$$\frac{(\alpha \int_{\psi_{HH}}^{m} (x+\theta_h) f(x) dx + (1-\alpha) \int_{\psi_{HL}}^{m} (x+\theta_l) f(x) dx)}{r/N + o_H} + i_L \frac{(\alpha \int_{\psi_{HL}}^{m} (x+\theta_h) f(x) dx + (1-\alpha) \int_{\psi_{LL}}^{m} (x+\theta_l) f(x) dx)}{r/N + o_L}$$
$$- \left(\frac{(\alpha \int_{\psi_{HH}^*}^{m} (x+\theta_h) f(x) dx}{r/N + o_H} + i_L \frac{(1-\alpha) \int_{\psi_{LL}^*}^{m} (x+\theta_l) f(x) dx)}{r/N + o_L}\right) \tag{20}$$

$$= \frac{(-\alpha \int_{\psi_{HH}^*}^{\psi_{HH}} (x+\theta_h) f(x) dx + (1-\alpha) \int_{\psi_{HL}}^{m} (x+\theta_l) f(x) dx)}{r/N + o_H} + i_L \frac{(\alpha \int_{\psi_{HL}}^{m} (x+\theta_h) f(x) dx - (1-\alpha) \int_{\psi_{LL}^*}^{\psi_{LL}} (x+\theta_l) f(x) dx)}{r/N + o_L}$$
$$> \frac{(-\alpha \int_{\psi_{HH}^*}^{\psi_{HH}} (\psi_{HH}+\theta_h) f(x) dx + (1-\alpha) \int_{\psi_{HL}}^{m} (\psi_{HL}+\theta_l) f(x) dx)}{r/N + o_H} + i_L \frac{(\alpha \int_{\psi_{HL}}^{m} (\psi_{HL}+\theta_h) f(x) dx - (1-\alpha) \int_{\psi_{LL}^*}^{\psi_{LL}} (\psi_{LL}+\theta_l) f(x) dx)}{r/N + o_L}$$

$= (1-\alpha)\int_{\psi_{HL}}^{m} f(x)dx\frac{\psi_{HL}+\theta_l-(\psi_{HH}+\theta_h)}{r/N+o_H}+\alpha\int_{\psi_{HL}}^{m} f(x)dx\frac{\psi_{HL}+\theta_h-(\psi_{LL}+\theta_l)}{r/N+o_L} > 0$. This holds if $m-\epsilon+\theta_l-(\psi_{HH}+\theta_h) > 0$ and $m-\epsilon+\theta_h-(\psi_{LL}+\theta_l) > 0$, which must hold for $\epsilon > 0$ sufficiently small, as F is continuous, $\alpha \in (0,1)$, and $\theta_H+\psi_{hh}^* < \theta_L+m$ and $\theta_L+\psi_{ll}^* < \theta_H+m$.

□

Lemma 6:

Proof. $HH1$ must be accepted as agents follow cutoff strategies and rejecting HH1 would yield an expected utility of 0, while accepting yields a strictly positive payoff. If $HL1$ is accepted, $LL1$ must also be accepted, as $c_h \geq c_l$. If $HL1$ is rejected, $LL1$ must be accepted by the same argument HH1 must be. $HL0$ and $LL0$ both generate a payoff of zero for at least one agent, while continuation values are strictly positive given that HH1 and LL1 must be accepted. □

Proposition 13:

Proof. There are 32 possible strategy profiles in this game–there are 2^4 potential acceptance strategies for Studs and two strategies for Duds for Duds. However, equilbrium requires a cutoff strategy. Additionally, $HH1$ must be accepted and $HL0$ and $LL0$ must be rejected. Thus, the only possible strategy profiles–represented by the mutual acceptance set–are $\{HH1, HH0, HL1, LL1\}$, $\{HH1, HH0, LL1\}$, $\{HH1, HL1, LL1\}$, and $\{HH1, LL1\}$. To simplify the comparison between the censored and uncensored search cases, we'd like $\{HH1, HH0, LL1\}$ to be the unique decentralized equilibrium, so we need to find conditions on the parameters to ensure profitable deviations for the other three cases. We now have

$$TSS = \frac{\alpha(I_{HH1}q(\theta_H+m)+I_{HH0}(1-q)\theta_H)+(1-\alpha)(I_{HL1}q(\theta_L+m)+I_{HL0}(1-q)\theta_L)}{o_H+r} + i_L\frac{\alpha(I_{HL1}q(\theta_H+m)+I_{HL0}(1-q)\theta_H)+(1-\alpha)(I_{LL1}q(\theta_L+m)+I_{LL0}(1-q)\theta_L)}{o_L+r} \quad (21)$$

, where I_d is the indicator for mutual acceqtance of match d. For $\{HH1, HH0, HL1, LL1\}$, we have $\alpha = 1-\frac{1}{1+\sqrt{q}}$, $o_H = \sqrt{q}$, and $o_L = q$. This yields $U_H = \frac{q+q^{3/2}+\sqrt{q}\theta_H}{\sqrt{q}+q+r+\sqrt{q}r}$. One potential deviation is to $\{HH1, HH0, LL1\}$, so ensuring deviation away from accepting high fit low types is strictly better for high types when holding α constant is sufficient. This yields $U_{H'} = \frac{\sqrt{q}(q+\theta_H)}{r+\sqrt{q}(1+r)}$, and $U_{H'} - U_H > 0$ if $r < \frac{\sqrt{q}(q-1+\theta_H)}{1+\sqrt{q}}$.

For $\{HH1, HL1, LL1\}$, we can exclude the case if $\theta_H > 1$.

For $\{HH1, LL1\}$, we have $\alpha = 1/2$, $o_H = q/2$, and $o_L = q/2$. This yields $U_H = \frac{q(1+\theta_H)}{q+2r}$. One potential deviation is to $\{HH1, HH0, LL1\}$, so ensuring deviation away from rejecting low fit high types is strictly better for high types when holding α constant is sufficient. This yields $U_{H'} = \frac{q+\theta_H}{1+2r}$, and $U_{H'} - U_H > 0$ if $r > \frac{q}{2\theta_H}$.

Finally, for $\{HH1, HH0, LL1\}$, we have $\alpha = 1 - \frac{1}{1+\sqrt{q}}$, $o_H = 1 - \frac{1}{1+\sqrt{q}}$, and $o_L = \frac{q}{1+\sqrt{q}}$. This yields $U_H = \frac{\sqrt{q}(q+\theta_H)}{r+\sqrt{q}(1+r)}$. We need there to be no profitable deviations, so we must exclude $\{HH1, LL1\}$ and $\{HH1, HH0, HL1, LL1\}$. $\{HH1, HL1, LL1\}$ is excluded based on in inequality above. Holding α constant, deviation to $\{HH1, LL1\}$ yields $U_{H'} = \frac{q^{3/2}(1+\theta_H)}{r+\sqrt{q}(q+r)}$, and $U_{H'} - U_H < 0$ if $r > \frac{q^{3/2}}{(1+\sqrt{q})\theta_H}$. Holding α constant, deviation to $\{HH1, HH0, HL1, LL1\}$ yields $U_{H'} = \frac{q+q^{3/2}+\sqrt{q}\theta_H}{\sqrt{q}+q+r+\sqrt{q}r}$, and $U_{H'} - U_H < 0$ if $r < \frac{\sqrt{q}(-1+q+\theta_H)}{1+\sqrt{q}}$.

The set of parameter vectors satisfying these constraints is not empty: $\theta_H = 1.3$, $q = 0.2$,$r = 0.13$ satisfies all of them. Thus, the platform can improve total surplus by censoring search under some parameters. However, these constraints are quite restrictive. $\square$

Proposition 14:

Proof. By Lemma 4, assume s=0 without loss of generality. By lemmas 2 and 3, there exists $\psi^* = (\psi^*_{HH},\psi^*_{HL}, \psi^*_{LH}, \psi^*_{LL})$ such that $\psi^*_{LH} = \psi^*_{HL}$ and ψ^* maximizes total surplus. Now let p be such that $0 \geq max\{max_{\psi\in\Psi}[EU_H[\psi, p]], max_{\psi\in\Psi}[EU_L[\psi, p]]\}$. Then every draw is accepted by every agent, regardless of agent strategy profiles. Now set $\psi_{\mathbf{c}} \equiv \psi^*$. Then agents accept every draw above the ψ^* cutoffs, and only receive those draws. Choose $\sigma_i = EU_i(\psi^*, p)$. Then $EU_i(\psi^*, p, f) = 0$.

$\square$

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Tables

Table 1: Frictionless Matching Simulations

	β	R^2	TSS_θ	TSS_ψ	$TSS = TSS_\theta + TSS_\psi$
Correlated ψ, 50 agents on each side, 40 iterations					
Non-transferable Utility, $a = 0$	0.79	0.61	25.00	45.81	70.81
Non-transferable Utility, $a = 1$	0.82	0.67	31.51	45.60	77.11
Transferable Utility, $a = 0$	-0.01	0.01	25.00	48.40	73.40
Transferable Utility, $a = 1$	0.78	0.59	30.93	47.52	78.46
Uncorrelated ψ, 50 agents on each side, 40 iterations					
Non-transferable Utility, $a = 0$	0.82	0.66	25.00	40.33	65.33
Non-transferable Utility, $a = 1$	0.87	0.75	31.85	40.06	71.91
Transferable Utility, $a = 0$	0.03	0.02	25.00	44.55	69.55
Transferable Utility, $a = 1$	0.83	0.70	31.81	42.69	74.50

Table 2: Optimal Values (CRM) with $F \sim U[0,1], i_L = 1, \theta_L = 1, \theta_H = 10$.

Utility	ψ_{HH}	ψ_{LL}	ψ_{HL}	r	α	p_H	p_L
12.9889	0.993087	0.99583	1	1.2346×10^{-6}	0.437139	-3.2367×10^{-6}	2.4289×10^{-6}
12.9006	0.937972	0.962629	1.0	0.0001	0.437001	-0.000253116	0.000196887
12.7259	0.828739	0.897044	1.0	0.00077	0.436731	-0.00195081	0.00152156
12.6071	0.754411	0.852579	1.0	0.0016	0.436548	-0.00404214	0.00315858
12.4683	0.667361	0.800675	1.0	0.0030	0.436332	-0.00748037	0.0058979
12.3346	-8.30175	0.7967	1.0	0.0051	0.310767	2.75809	0.854039
12.2368	-8.34786	0.727159	0.69825	0.0081	0.343117	3.06221	1.09646
12.1236	-8.40321	0.649491	0.652146	0.012	0.371874	3.34222	1.39004
11.9942	-8.46845	0.564259	0.596788	0.018	0.397629	3.60453	1.74287
11.8483	-8.54382	0.472208	0.531551	0.026	0.420791	3.95393	2.16104
11.6858	-8.62925	0.374179	0.456182	0.035	0.44168	4.09441	2.64872
11.5071	-8.72472	0.271013	0.370748	0.047	0.46057	4.3298	3.20946
11.3128	-8.82951	0.163734	0.275285	0.062	0.477664	4.56292	3.84295
11.1038	-8.94326	0.0531883	0.170495	0.081	0.4931169	4.79709	4.54967
10.6232	-9	6.4099×10^{-9}	5.3382×10^{-7}	0.13	0.5	4.8704	4.8704
10.0206	-9	2.1392×10^{-9}	5.8450×10^{-7}	0.20	0.5	4.80247	4.80247
9.30806	-9	4.8821×10^{-8}	4.1757×10^{-8}	0.29	0.5	4.71079	4.71079
6.28125	-9	0	4.9856×10^{-8}	1.0	0.577386	4.69647	4.69647
2.4375	-9	0	1.7323×10^{-7}	4.0	0.497393	0.976537	0.976537

Table 3: Optimal Values (LRM) with $F \sim U[0,1], i_L = 1, \theta_L = 1, \theta_H = 10$.

Utility	ψ_{HH}	ψ_{LL}	ψ_{HL}	r	α	p_H	p_L
12.9989	0.99943	0.999817	1	0.00000123	0.361776	-3.74 $\times 10^{-8}$	-3.83 $\times 10^{-9}$
12.9789	0.989354	0.996586	1	0.0001	0.361551	-0.0000130906	-1.347 $\times 10^{-6}$
12.9179	0.958548	0.986735	1	0.00077	0.361304	-0.000198876	-0.0000203987
12.8669	0.932753	0.978519	1	0.0016	0.361098	-0.00052484	-0.000536269
12.8	0.898878	0.96774	1	0.002964	0.360826	-0.00118817	-0.000121101
12.7158	0.856194	0.95431	1	0.00567	0.360478	-0.00240974	-0.000244527
12.6131	0.80408	0.937978	1	0.0081	0.36006	-0.00448835	-0.000452905
12.4911	0.74203	0.918691	1	0.01235	0.359556	-0.00781437	-0.000783347
12.3491	0.669651	0.896408	1	0.018075	0.358968	-0.0128782	-0.00128098
12.1867	0.586675	0.87115	1	0.256	0.35829	-0.0202767	-0.00199907
12.0234	-8.2974	0.744358	0.702597	0.03526	0.335817	2.97117	0.914171
11.8743	-8.33934	0.684187	0.660659	0.047427	0.359784	3.1995	1.12435
11.7044	-8.3905	0.617436	0.609499	0.0625	0.382151	3.42066	1.38295
11.5134	-8.45143	0.544189	0.548568	0.0809	0.403034	3.63694	1.69696
11.0691	-8.60322	0.379101	0.396784	0.1296	0.440707	4.06307	2.51239
10.5481	-8.7924	0.192178	0.2076	0.19753	0.473349	4.49165	3.59388
9.96265	-9	1.12993	7.39 $\times 10^{-8}$	0.2892	0.5	4.90724	4.85409
7.02944	-9	5.6692	1.03 $\times 10^{-8}$	1	0.5	4.7388	4.54934
3.13445	-9	1.6097	4.752 $\times 10^{-8}$	4	0.5	4.41421	3.75736